SEMINAR

RUNNING TIME AND CIRCUIT COMPLEXITY

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THEOREM

If $L \subseteq \{0,1\}^*$ can be recognized by a TM whose running time is bounded above by $T(n)$, then the corresponding sequence $\{f_n\}$ of Boolean functions has the property that the n^{th} function can be realized using $O(T(n)^2)$ gates.

TM M = $(Q, q_0, Q_f, \Sigma, \partial)$

- Q is the set of states.
- q_0 is the initial state.
- $Q_f \subseteq Q$ is the set of accepting states.
- Σ is the finite alphabet.
- $\partial : Q \times \Sigma \rightarrow Q \times \Sigma \times \{L,R\}$ be the transition function.

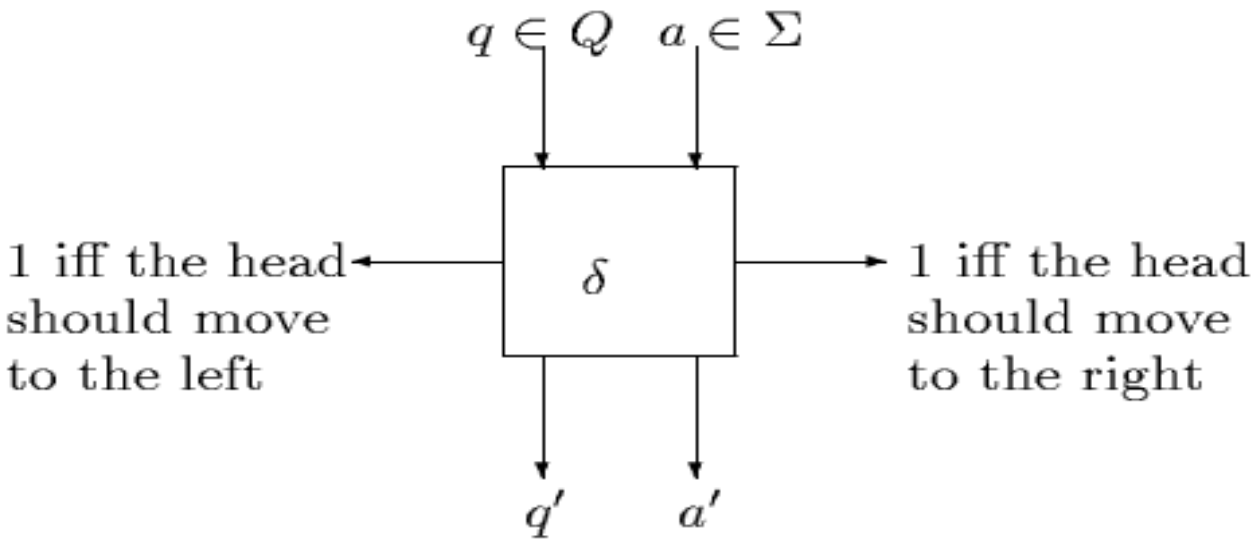
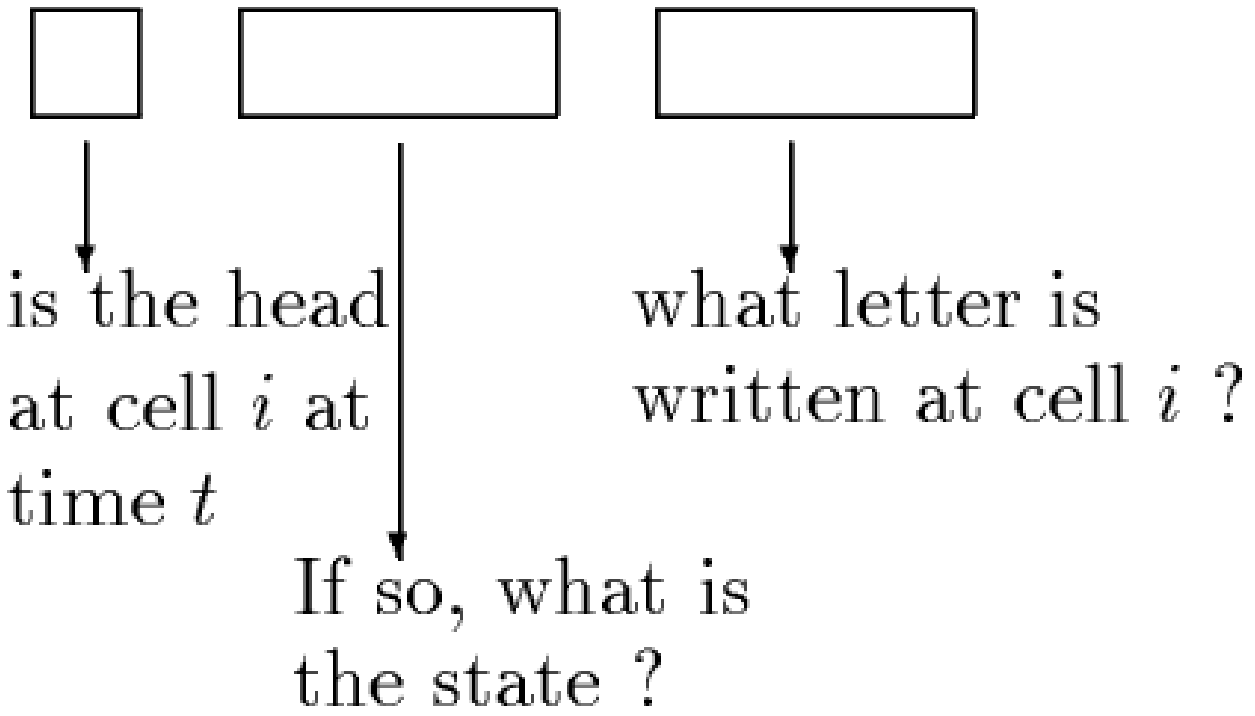


Figure 3.1: Inputs and outputs of the δ circuit

Input : Boolean encoding of the initial state q and the symbol a.

Output: New state q', output symbol a' and L or R bit is set based on the movement of head.

1 000 1


is the head at cell i at time t
If so, what is the state ?
what letter is written at cell i ?

TABLEAU

1 001 1	0 000 1	0 000 0	Π	Π

- For the first row:

  The data parts of the first n cells in the first row are filled in with the input X, leaving the remaining cells blank. The head-present bit of the first cell is set and the state of the first cell is set to q0.

To compute the values of subsequent rows :

The values of a cell in the ith row depend on the three cells directly above it which can be computed by a constant sized circuit.

If the head was not present in the three cells directly above, then the head state and the content cannot change.

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0 000 0	0 000 0	0 000 1

0 000 0

1 101 0	0 000 0	0 000 1
	1 111 0	

ACCEPTANCE CRITERION

- If the state of the cell with the head is an accepting state, the circuit outputs 1,0 otherwise.
- No. of columns = S(n)
- No. of rows = T(n)
- Size =O(T(n).S(n)) = $O(T(n)^2)$ Since(T(n)>=S(n)).
- Schnorr Pipenger Theorem : O(T(n).log(T(n)))

TURING MACHINE THAT TAKE ADVICE

- A TM that takes advice has the property that for every input size, the input to the TM is the input word “x” and an advice string “w” which depends on the length of the input.
- The class of languages decidable by time-T(n) TMs with a(n) bits of advice is denoted by DTIME(T(n))/a(n).

- P/poly is the class of languages which can be recognized by TMs that take advice in time polynomial in length of the original input x.
- The poly-sized circuit recognizing the inputs of size n can be encoded to form the advice string for input of length n. The TM can then simply evaluate the value of circuit on input string x.