Assignment II

Q1. Let L be the regular language generated by the regular expression $(a^*bb + b^*aa)^*$ over $\Sigma = \{a, b\}$. Let $\overline{L}$ denote the complement of L.

1. Construct a minimum DFA for $\overline{L}$.

2. Write down a regular expression for $\overline{L}$.

3. How many Myhill Nerode equivalence classes does L have? Write down one string from each of the Myhill Nerode equivalence classes of L.

4. How many Myhill Nerode equivalence classes does $\overline{L}$ have? Write down one string from each of the Myhill Nerode equivalence classes of $\overline{L}$.

Q2. Prove that the language $L = \{a^n b^m : n \neq m\}$ is not regular by showing that there are infinitely many Myhill Nerode equivalence classes. (Here is another way to prove. Consider $L' = \overline{L} \cap a^*b^*$. Suppose L was regular, then L' should have been regular (why?), which would lead to a contradiction - (how?)).

Q3. Show that the language $L = \{ab^jc^j : j \geq 0\} \cup \{a^ib^jc^k : i, j, k \geq 0 \text{ and } i \neq 1\}$ satisfies the conditions of the Pumping Lemma, but is not regular. This shows that even though all regular languages salsify Pumping Lemma conditions, not every language that satisfies the conditions of the Pumping Lemma is regular.

Q4. Show that the language of all strings over $\{a, b\}$ with equal number of $a's$ and $b's$ is not regular.

Q5. Is the class of regular languages closed under infinite union? Prove/Disprove.

Q6. Let L_1, L_2 be two languages. Define $L_1/L_2 = \{x : \exists y \in L_2 \text{ such that } xy \in L_1\}$. That is L_1/L_2 consists of those strings x having the property that we can find a string $y \in L_2$ such that if we suffix y to x, we get a string in L_1. Show that L_1/L_2 is regular.

Q7. If $f(n)$ be a constructible function. Can we always say that DTime$(f(n)) \subseteq$ NTime$(f(n))$? Show that DTime$(f(n))$ =coDTime$(f(n))$, where coDTime$(f(n)) = \{L : \overline{L} \in \text{DTime}(f(n))\}$.

Q8. Let $f(n) \geq n$ be a constructible function. Show that NTime$(f(n)) \subseteq$ DTime$(2^{O(f(n))})$.

Q9. Show that a single tape Turing machine can accept the language $L = \{a^nb^n : n \geq 0\}$ in $O(n \log n)$ steps. That is, show that $L \in \text{DTime}(n \log n)$. (In such questions, you don't have to write down the transitions of the Turing machine, but explain the logic of how the machine can accept the language within the required time bounds.)