Assignment I

1. Balls are randomly (and independently) thrown into m bins kept at a place.
    1. What is the expected number of balls to be thrown before the first bin gets a ball?
    2. After having thrown n balls, what is the expected number of balls in the first bin?
    3. What is the expected number of throws before the first bin gets a second ball? (That is there is a collision in the first bin - note the relationship with hashing.)
    4. What is expected number of throws before at least one bin gets two balls? (number of insertions before the first collision).
    5. What is the expected number of throws before at least two bins get a ball? (Hint: Let X be a random variable denoting the number of throws after the first throw necessary for a ball to land in a bin different from the bin into which the first throw landed.).
    6. Generalize the above question - what is the expected number of throws before k bins have at least one ball.
    7. What is the expected number of throws before every bin gets a ball?.
2. Suppose you keep tossing a coin (with probability of head p) independently till you see exactly n heads.
    1. For $i \geq n$, what is the probability that i throws are required to get n heads?
    2. Let X denote the random variable denoting the number of throws before you see n heads. Find $E(X) = \sum_i i.Pr(X = i)$.
    3. Let $X_1, X_2, .X_n$ denote random variables such that X_1 denote the number of throws required for the first head to appear, X_{i+1} denote the number of throws required after the appearance of the i^{th} head to obtain the $(i+1)^{th}$ head, $i \geq 1$. Show that $E(X)$ of the previous question can be evaluated in a simpler way using these random variables using linearity of expectation.
3. Suppose X is a random variable that takes values from the set $\{0, 1, 2, ...\}$. Show that $E(X) = Pr(X \geq 1) + Pr(X \geq 1) + Pr(X \geq 2) + ...$
4. Suppose there are 2 bins B_1 and B_2. B_1 contains 2 White and 3 Red balls. B_2 contains 3 White and 4 Red balls. Suppose you pick B_1 or B_2 with probability 1/3 or 2/3 and then a ball randomly from the bin.
    1. Describe all points in the sample space of the experiment.
    2. Suppose you got a White ball, what is the conditional probability that the bin was B_2?
5. Suppose we choose a number between 1 and 4 at random as our first step. If you get 1, we will stop. Otherwise, if we get $i \in \{2, 3, 4\}$, we will choose an integer between 1 and $i - 1$ at random again as our second step. We keep doing this till we choose 1.
    1. Enumerate the sample space of this experiment.
    2. Suppose you got a 1 in the second step, what is the (conditional) probability that you picked 4 in the first step.
6. Mr. Markov is waiting the Poissonville bus stop where buses arrive with Probability 1/6 uniformly and independently at the strike of each minute. Assume Markov has arrived at the bus stop exactly at the strike of a minute.
    1. What is the average wait time Mr. Markov spends in the bus stop?
    2. Suppose a bus just left the previous second before Mr. Markov arrived, what is the expected wait time of Mr. Markov conditioned on this fact?
    3. Suppose Mr. Chebyshev arrive exactly 3 minutes after the arrival of Mr. Markov. What is the probability that they get the same bus?