1. Is it true that coNP$\subseteq$ PSPACE? Show that NTIME(n) is a proper subset of PSPACE. □

2. Is it true that NTIME($f(n)$)$\subseteq$ DSPACE($f^2(n)$) for $f(n) \geq \log n$. □

3. Show that 2SAT is in **NL**. Show that if 3SAT is in **NL** then **P**=**NP**. □

4. Show that the problem of checking whether a given n bit number is prime is in coNTIME(n^2). □

5. Show that if **P**=**NP** then every language in **P** except just two languages are NP-complete. Which are those two languages? (Hint: One of the languages is the empty set). □

6. Define **polyL** as $\cup DSPACE(\log^i n)$. Steve's class **SC** is defined as the class of languages that can be decided by deterministic machines that run in polynomial time and $\log^i n$ for some $i \geq 0$. Why does it NOT follow from Savitch's theorem that **SC**=**PolyL**? Is **SC** same as **polyL**$\cap$ **P**? □

7. Recall that in the proof of the hierarchy theorem showing that for any constructable function $f(n) \geq \log n$, there exists a language $L \in DSPACE(f(n)) \setminus DSPACE(g(n))$ for any $g(n) \in o(f(n))$, we defined the langauge L=$\{$x=M0*: M is a TM and rejects M in atmost $f(|x|)$ steps $\}$. What goes wrong with the proof if instead of L, we the langauge L'=$\{$ M : M is a TM and M rejects M in atmost $f(|M|)$ steps$\}$ is used in the proof? □

8. We have defined **NP** as $\bigcup_{i\geq 0} NTIME(n^i)$. Here is an alternative definition. We define **NP** to be the class of all langauges $L \subseteq \Sigma^*$ such that there is a deterministic Turing machine M that runs in time polynomial in the size of its input and an integer k such that if $x \in L$ then there exists $y \in \Sigma^*$ with $|y| \leq |x|^k$ and $M(x, y) = 1$ whereas if $x \notin L$, for all $y \in \Sigma^*$ $M(x, y) = 0$. This is the "certificate" characterization of the class **NP** Show that the two definitions are equivalent. (ie., show that for any language L in **NP**, such machine M exists and conversly, given M, we can contruct a polynomial time NDTM M' for accepting $\{x|M(x, y) = 1\}$. □

9. Derive a similar characteristic for **co-NP**. □

10. Show that if L is NP-complete then its complement is co-NP complete. □

11. Prove that **P**$\subseteq$ **NP**$\cap$ **co-NP**. □

12. Show that SAT$\preceq_m^p$3SAT. CLIQUE$\preceq_m^p$VERTEX COVER and VERTEX COVER$\preceq_m^p$INDEPENDENT SET. □

13. Assuming NP-completeness of the Directed Hamiltonian Path (DHP) (called $HAMPATH$ in Sipser, Theorem 7.35), Show that the following problems are NP-complete: □

    1. DHC: Given a directed graph G, does it contain a (directed) Hamiltonian cycle?
    2. UHP: Given an (undirected) graph G, does it contain a Hamiltonian path?
    3. UHC: Given an (undirected) graph G, does it contain a Hamiltonian cycle?

14. A directed graph is *strongly connected* if every two nodes are connected by a directed path in each direction. Let STR-CON=$\{G|G$ is a strongly connected graph$\}$. Show that the problem is NL-complete. □

15. Show that the problem of checking whether a graph is bipartite is is NL. (You need to know a characterization of bipartite graphs to solve this — look at your graph theory text.) □

16. Let $L \in NTIME(2^{n^c})$ for some constant $c > 0$. Define the language $L_{pad} = \{(x, 1^z) : z = 2^{|x|^c}\}$. Show that $L_{pad} \in NP$ Hence show that if P=NP then NEXP=EXP. □

17. Show that for any language L, L$\in$DTIME(n^k) if and only if for any L'=$\{(x, 1^z)$ where, $x \in L$, $z = |x|^k$ $\}$ is in DTIME(n). Hence show that P$\neq$ PSPACE. □

18. Show that there exists an oracle C such that $NP^C \neq coNP^C$. □