Name and Roll No.: ______________________________

1. Is $\mathcal{Z}$ a vector space over $\mathcal{Z}_2$? If yes, what is the dimension of $\mathcal{Z}$ over $\mathcal{Z}_2$? If not, which vector space property is violated? [3]

2. Consider the map T from $\mathcal{R}^2$ to itself sending the point $[x, y]^T$ to the point $[x, -y]$. What is $Rank(T)$? [3]

3. In the vector space $\mathcal{C}$ over $\mathcal{R}$, what are the **cordinates** of the number $1 + 2i$ with respect to basis $\{i + i, i - i\}$? [3]

4. Let $S \subseteq V(F)$ be a maximal linearly independent set (that is, S is linearly independent, but adding any other vector in V to S would made S linearly dependent). Can you say that S is a basis for V? Justify your answer. [3]

5. Let $\mathcal{Q}[x]$ be the ring of polynomials with coefficients in Q as a vector space over the field $\mathcal{Q}$. Consider the map Φ from $\mathcal{Q}[x]$ to Q defined by $\Phi(f) = f(0)$. That is, the map that evaluates the polynomial at zero. Is Φ a homomorphism? What is $ker(\Phi)$? What is $img(\Phi)$? [3]

6. For what integer values of m can there be a ring homomorphism from Z_{10} to Z_m? Justify your answer. [3]

7. In the group $\mathcal{Z}_{10}$ (with addition $\mod 10$). Consider the subgroup $H = \{0, 5\}$. list all the cosets defined by H? [3]

8. Consider the ring $\mathcal{Z}_4 \times \mathcal{Z}_6$ consisting of tuples (a, b) with $a \in \mathcal{Z}_4$ and $b \in \mathcal{Z}_6$. Suppose (a, b) and (a', b') are tuples in $\mathcal{Z}_4 \times \mathcal{Z}_6$ then we defined $(a, b) + (a', b')$ to be $(a + a' \mod 4, b + b' \mod 6)$. Similarly mulitplication gives $(aa' \mod 4, bb' \mod 6)$. $\mathcal{Z}_4 \times \mathcal{Z}_6$ is a ring with these operations and is called the product ring of $\mathcal{Z}_4$ and $\mathcal{Z}_6$. Consider the map f from $\mathcal{Z}_{24}$ to $\mathcal{Z}_4 \times \mathcal{Z}_6$ sending the element x in $\mathcal{Z}_{24}$ to the tuple $(x \mod 4, x \mod 6)$. The map f is a ring homomorphism. [4 x 2]

   - What is $ker(f)$ and $img(f)$?

   - What is $ker(f)$ and $img(f)$ if the map was from $\mathcal{Z}_{15}$ to $\mathcal{Z}_3 \times \mathcal{Z}_5$?