Assignment II

1. Construct DPDAs accepting on empty stack and final state for the language $\{a^ib^jc^{i+j} : i,j \geq 0\}$. Construct a CFG corresponding to your PDA based on the method explained in the class.

2. Show that the above language is context free using the Chomsky Schutzenberger theorem (find a parenthesis language, regular language over the alphabet of parenthesis and a homomorphism such that the given the language is the homomorphic image of the parenthesis language intersection the regular language)

3. Let $M = (Q_1, \Sigma, \Gamma, \delta_1, q_0, F_1)$ be a PDA (that accepts on final states) and $R = (Q_2, \Sigma, \delta_2, p_0, F_2)$ be a FSM. Consider the following PDA $N = (Q, \Sigma, \Gamma, \delta, (q_0, p_0), F)$ where $Q = Q_1 \times Q_2$ and δ defined as follows: if $(q', \alpha) \in \delta_1(q, a, X)$ in M and $\delta_2(p, a) = p'$ in R, then we include $((q', p'), a, \alpha)$ in $\delta((q, p), a, A)$ for any $a \in \Sigma \cup \{\epsilon\}$, $A \in \Gamma$ and $\alpha \in (\Sigma \cup \Gamma)^*$. Define the set F appropriately so that the resultant PDA N accepts $L(M) \cap L(R)$. Prove formally that $L(N) = L(M) \cap L(R)$. This construction shows that the intersection of a context free language with a regular language is context free.

4. Construct PDA for the language over $\{a, b\}$ with equal number of $a's$ and $b's$. Construct the product automation with DFA for a^*b^* using the method described above.

5. Use the Pumping lemma to show that the language $\{a^nb^{2n}c^n : n \geq 0\}$ is not a CFL. Hence show that the language $L = \{ww : w \in \{a, b\}^*\}$ is not context free.

6. Show that the complement of the language $L = \{ww : w \in \{a, b\}^*\}$ is context free. Hence L is a example for a language which is not a CFL, but whose complement is context free. Note that since DCFLs are closed under complementation, it is impossible for the complement of L to be a DCFL.

7. Let $h : \Sigma \longrightarrow \Gamma^*$ be a homomorphism and L be context free. Prove that $h(L)$ is context free. (Think in terms of grammar). Thus context free languages are closed under homomorphisms. Hence show that the language $L' = \{a^nb^nc^nd^n : n \geq 0\}$ is not context free.

8. Show that the complement of the language $\{a^nb^n : n \geq 0\}$ is context free. (Using closure properties would be easier than directly try to construct PDA/CFG. Ability to use such indirect methods will be crucial for the examination.)

9. Construct Turing machines for the languages $L = \{a^nb^nc^n : n \geq 0\}$ and $L' = \{ww : w \in \{a, b\}^*\}$. Analyze the time complexity of your constructions.

10. Let $N = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA accepting some language L over Σ^*. Construct a single tape Turing machine $M = (Q\Sigma, \Gamma', \delta', q_0', q_f, q_r)$ that simulates N. (note, the TM machine must first move to the left end read the input from left to right). Write down the transitions of the Turing machine and analyze the time complexity of your simulation. Construct a two-tape Turing machine that simulates the PDA in $O(n)$ time.

11. Recall the problems $CLIQUE = \{(G, k)$: G is a graph containing a clique of the input size $k \geq 0\}$, $VC = \{(G, k)$: G is a graph with a vertex cover of the given size $k \geq 0\}$ and $IS = \{(G, k)$: G is a graph with an independent set of the given size $k \geq 0\}$. Show that all these problems are polynomial time many one reducible to each other by providing appropriate reduction algorithms.

12 The subgraph isomorphism problem $SI = \{(G_1, G_2)$: G_1 contains a subgraph isomorphic to $G_2\}$. Give a reduction algorithm to show that $CLIQUE \preceq_m^p SI$