Final Exam Part I **Theory of Computation** **Total: 20 Marks**

Name and Roll No.: ________________________________

1. Let $L = \{M : L(M)$ is Turing acceptable $\}$ Is L Turing decidable? Justify. $\boxed{2}$
   Soln: For any machine M, $L(M)$ is trivially Turing acceptable (M itself accepts $L(M)$ by definition!). Thus L is essentially the set of all Turing machines. But this is easily decidable as it is easy to check whether the input represents the encoding of a valid Turing machine.

2. Recall that $coNP = \{L : \overline{L} \in NP\}$. Suppose $NP \neq coNP$, can we conclude $P = NP$? Justify. $\boxed{2}$
   Soln: The question is equivalent to showing that if $P = NP$ then $NP = coNP$. But if $NP = P$, then P now being a deterministic class, is closed under complementation (just swap accepting state and rejecting states). That is, $P = coP$. Hence $NP = P = coP = coNP$.

3. Is it true that $NL \neq PSPACE$? Justify. $\boxed{2}$
   Soln: $NL \subseteq L^2$ by Savitch Theorem. But L^2 is a strict subset of $Dspace(n^k)$ for any k by Hierarchy theorem. Hence $P \neq PSPACE$.

4. For a time/space) complexity function $f(n)$, define $coNTIME(f(n))/coNSPACE(f(n))$ as the set of all languages whose complements are in $NTIME(f(n))/NSPACE(f(n))$. (Justify your answers:) $\boxed{2+2}$

   (a) Give a function $g(n)$, asymptotically as small as possible, such that $coNTIME(f(n)) \subseteq Dspace(g(n))$.
   Soln: $coNTIME(f(n)) \subseteq coNSPACE(f(n)) \subseteq Dspace(f^2(n))$ by Savitch and the fact that deterministic classes are closed under complementation.

   (b) Give a function $h(n)$, asymptotically as small as possible, such that $coNTIME(f(n)) \subseteq NTime(h(n))$.
   Soln: $coNTIME(f(n) \subseteq Dtime(2^{f(n)})$. (again using the fact that $NTIME(f(n) \subseteq Dtime(2^{f(n)})$ and the RHS is closed under complementation).

5. Suppose it is true that $Nspace(f(n)) = Dspace(f^2(n))$. $\boxed{2+2+2}$

   (a) Can you conclude the $L = P$ or $L \neq P$. (Here $L = Dspace(\log n)$). Justify your answer.
   Soln: The hypothesis implies that $NL = L^2$. But $L \neq L^2$ by space hierarchy theorem. Also we know that $NL \subseteq P$. Hence $L \neq NL \subseteq P$.

   (b) Can you conclude that $L = NL$ or $L \neq NL$? Justify your answer.
   Soln: Answered in the previous question.

   (c) Can you conclude that $NL = NP$ or $NL \neq NP$? Justify your answer.
   Soln: Nothing can really be inferred about this from what is given!

6. Suppose a language $L \in NP$ is proved to be EXP-complete (Justify your answers) $\boxed{2+2}$

   (a) Is it necessary that L is NP complete?
   Soln: If an NP-complete problem is EXP-complete, then $NP = PSPACE = EXP$ and every EXP complete problem is NP-complete as well (and $PSPACE$ which was between the two classes get sandwiched).

   (b) Can we conclude that $P = PSPACE$ or $P \neq PSPACE$?
   Soln: Since $P \neq EXP$ by time hierarchy theorem, and the hypothesis implies $NP = PSPACE = EXP$, it follows that $P \neq PSPACE$.