Mid I **Theory of Computation** **Total: 30 Marks**

Name and Roll No.: ____________________

1. Suppose $L_1, L_2, \ldots$ is a countable infinite family of regular languages over alphabet $\{0, 1\}$. Can we say that $\bigcup_{i=1}^{\infty} L_i$ is regular? (Answer Yes/No. If Yes, Prove. If No, Give counter-example.) $\boxed{1}$

   Soln: No. Consider the family of languages $L_0 = \{\epsilon\}$, $L_1 = \{ab\}$, $L_2 = \{aabb\}$, $L_3 = \{aaabbb\}$ etc. Each language in the sequence is regular as it consists of just one string. However, their union is $\{a^n b^n : n \geq 0\}$ which is not regular. (In fact, any language can be expressed as a countable union of singletons.) Note that the closure of regular languages under finite union does not imply closure under infinite union.

2. Let M_1 and M_2 be finite state machines over $\{0, 1\}$ with state sets Q_1, Q_2 and F_1, F_2 as the set of final states. Suppose you want the product automation $M_1 \times M_2$ to accept $L(M_1) - L(M_2)$, what must be the final states? Justify your answer in one sentence. $\boxed{2}$

   Soln: $F_1 \times (Q_2 - F_2)$. That is all strings leading to a final state of M_1 and a non-final state of M_2 must be accepted.

3. In the proof of the pumping lemma, it was shown that for every CFL L, there exists an n such that for all $z \in L$ with $|z| \geq n$, $\exists u, x, w, y, v$ such that $z = uxwyv$ and $ux^i wy^i z \in L \; \forall \; i \geq 0$. Is it true that $|uxwy| \leq n$; i.e., the repetition pattern must be spotted before the first n symbols of z? (Answer YES/NO first. If Yes, give Proof. If No, give give counter-example). $\boxed{3}$

   Soln: No. For instace, consider the language $\{a^i b^i | i \geq 0\}$. Let n be the constant specified in the Pumping Lemma and consider the string $a^n b^n$ in the language. It is impossible to pump (repeat) any substring within the first n positions and get a string in the language.

4. Is the language $L = \{a^i b^j : i = j \text{ if } i \leq 3\}$ regular? (Answer YES or NO. If the answer is Yes, give a DFA. Otherwise give a proof that L is not regular by giving an infinite number of Myhill Nerode inequivalent strings). $\boxed{4}$

   Soln: Yes. Here is a regular expression for the language (from which one can construct a DFA). $\epsilon + ab + aabb + aaabbb + aaaaa^* b^*$.

5. Consider the grammar $G : S \longrightarrow Aa|Bb|Sab, A \longrightarrow Aa|a, B \longrightarrow Bb|b$. Construct a finite state machine that accepts $L^R(G)$, the language consisting of all strings in $L(G)$ written in reverse. (Hint: What is the grammar for $L^R(G)$?) Answer on the reverse side. $\boxed{5}$

   Soln: If we reverse the right side of each production, we will get a right linear grammar for $L^R(G)$. The method of constructing a DFA from a right linear grammar was posed in Assignment I.