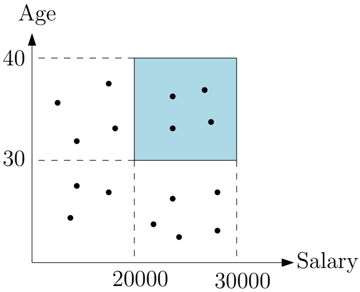
# Range searching

Aritra Banik*
National Institute of Science Education and Research

Summer School on Graph Theory and Graph Algorithm at NIT Calicut

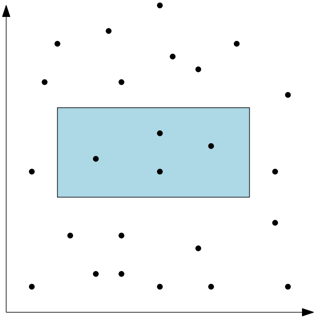
*Slide ideas borrowed from Marc van Kreveld and Subhash Suri

# Range query



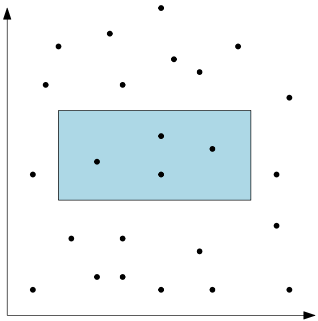

- A range query is a common database operation that retrieves all records where some value is between an upper and lower boundary.
- Range query: Asks for the objects whose coordinates lie in a specified query range (interval)

# Range searching

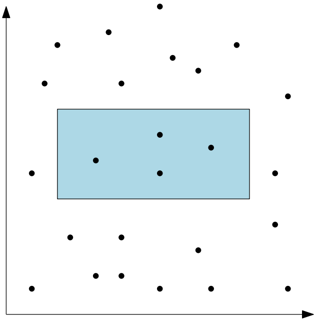


- Range Searching: Process a set of given data points efficiently such that given a range window set of points inside the range can e reported "QUICKLY".
- Time-Space tradeoff: the more we preprocess and store, the faster we can solve a query.
- A (search) data structure has a storage requirement, a query time, and a construction time (and an update time)

# Range searching



- Construction time $O(1)$: query time??
- Objective is sub linear query time.



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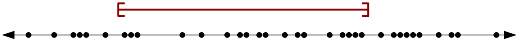
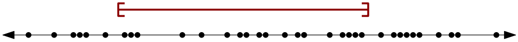


- 1D range query problem: Preprocess a set of $n$ points on the real line such that the ones inside a 1D query range (interval) can be reported fast.
- The points $p_1 \ldots p_n$ are known beforehand, the query $[x, y]$ arises at run time.
- A solution to a query problem is a data structure description, a query algorithm, and a construction algorithm.

# 1D range query problem



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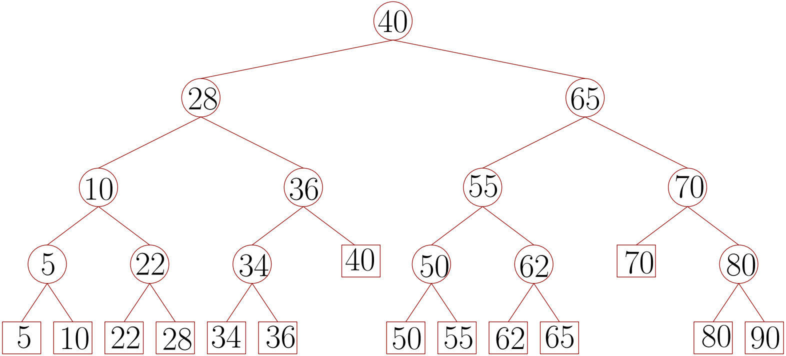
The Data Structure: Balanced binary search trees




- Query $[34, 80]$
- Search path for 34.
- Search path for 80.

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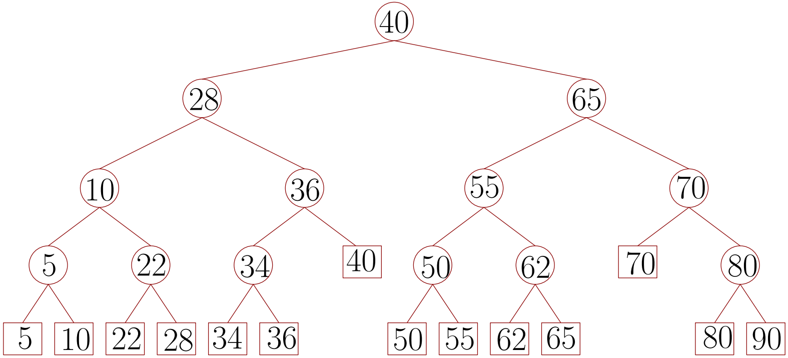


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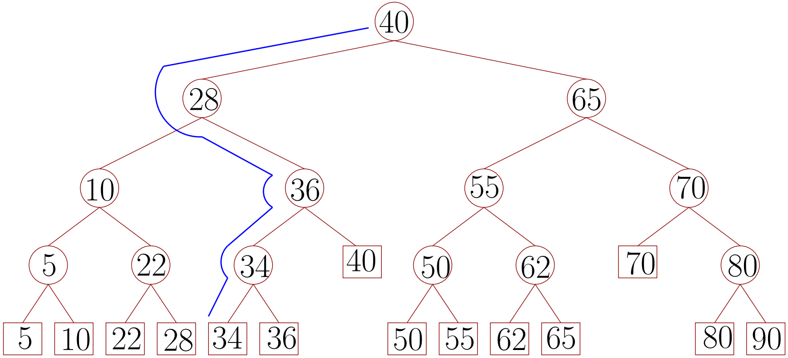


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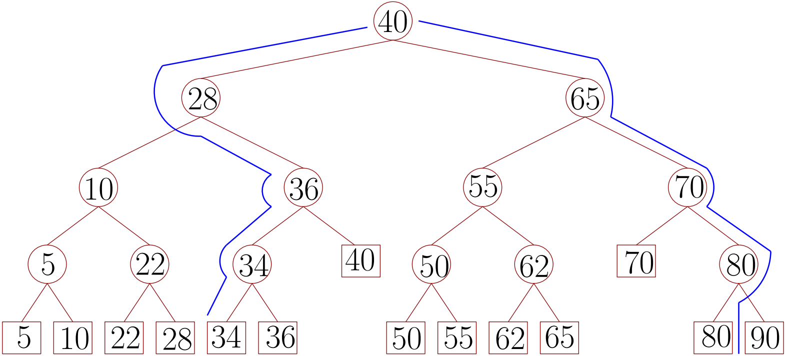


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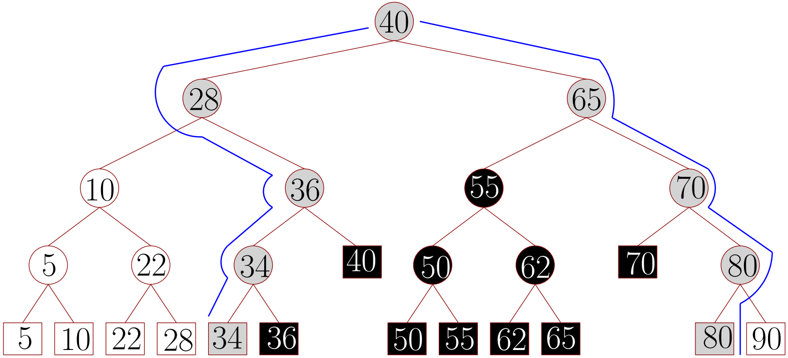
Three types of nodes for a given query:




- **White nodes:** never visited by the query
- **Grey nodes:** visited by the query, unclear if they lead to output
- **Black nodes:** Visited by the query, whole subtree is output

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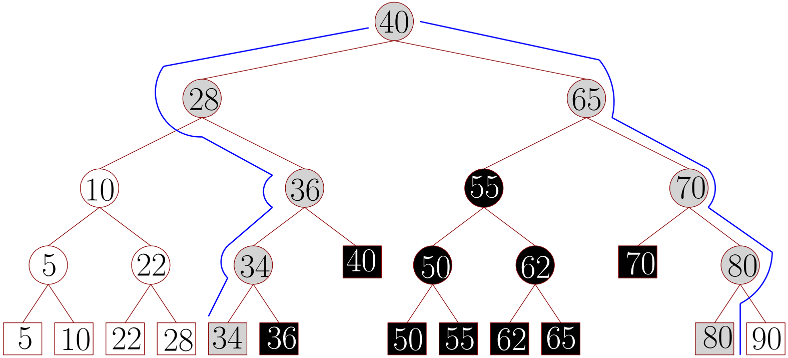


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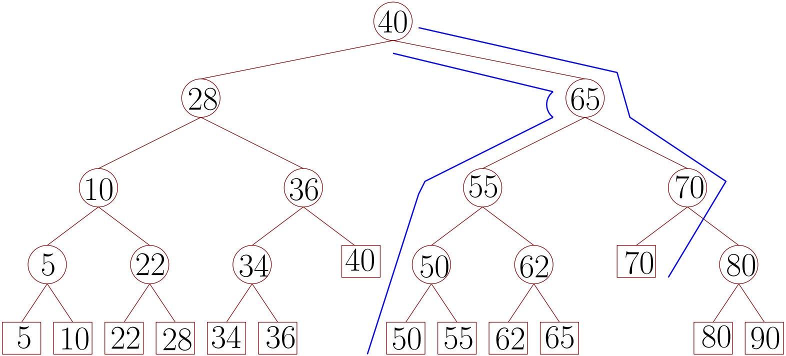
Find a split node [50 – 70]

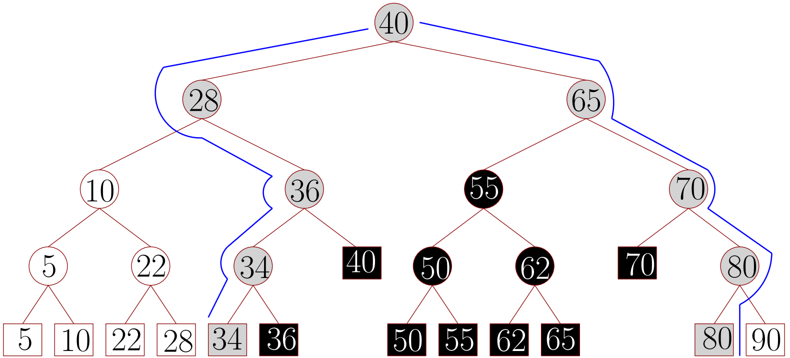


# Algorithm

**Algorithm 1** 1DRangeQuery($T$, $[x : y]$)

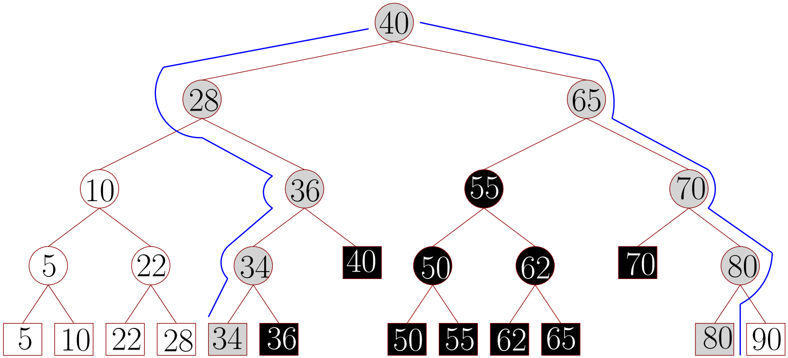
$v_{split} \leftarrow$ FindSplitNode($T, x, y$)
**if** $v_{split}$ is a leaf **then**
  Check if the point in $v_{split}$ must be reported.
**else**
  $v \leftarrow lc(v_{split})$
  **while** $v$ v is not a leaf **do**
    **if** $x \leq value(v)$ **then**
      ReportSubtree($rc(v)$)
      $v \leftarrow lc(v)$
    **else**
      $v \leftarrow rc(v)$
    **end if**
  **end while**
  $v \leftarrow rc(v_{split})$
  Similarly, follow the path to $y$
**end if**

# Runtime




- White nodes: never visited by the query; no time spent
- Grey nodes: visited by the query, unclear if they lead to output; time determines dependency on $n$
- Black nodes: visited by the query, whole subtree is output; time determines dependency on $k$, the output size

# Runtime



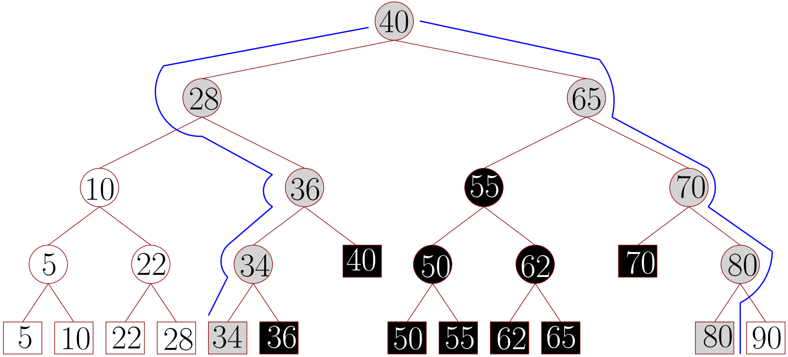

- Grey nodes: they occur on only two paths in the tree, and since the tree is balanced, its depth is $O(\log n)$
- Black nodes: Charged on output

The time spent at each node is $O(1) \Rightarrow O(\log n + k)$ query time

# Storage requirement and preprocessing

- A (balanced) binary search tree storing n points uses $O(n)$ storage
- A balanced binary search tree storing $n$ points can be built in $O(n)$ time after sorting, so in $O(n \log n)$ time overall (or by repeated insertion in $O(n \log n)$ time)

A 1-dimensional range tree for range counting queries




**Theorem**

*A set of n points on the real line can be preprocessed in $O(n \log n)$ time into a data structure of $O(n)$ size so that any range counting queries can be answered in $O(\log n)$ time*

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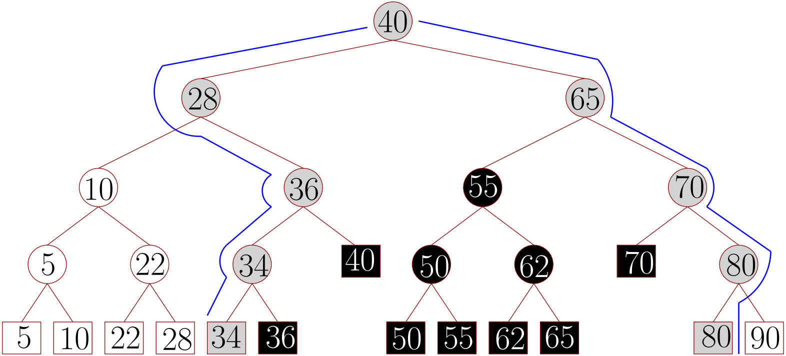


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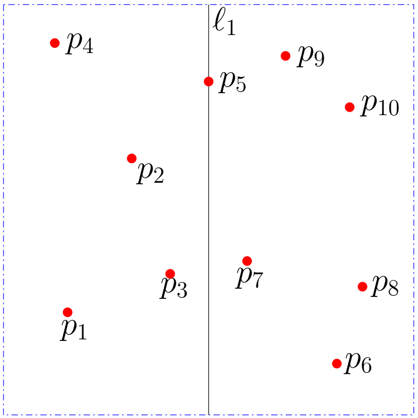
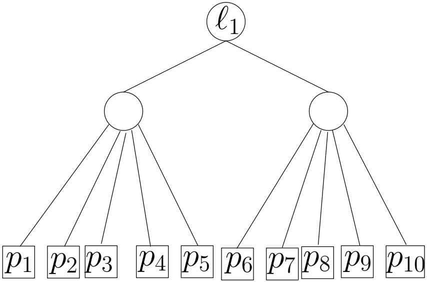
# 2D Range queries

Kd-trees, the idea:

- Split the point set alternatingly by x-coordinate and by y-coordinate
- Split by x-coordinate: split by a vertical line that has half the points left and half right
- Split by y-coordinate: split by a horizontal line that has half the points below and half above

# Kd-tree Construction



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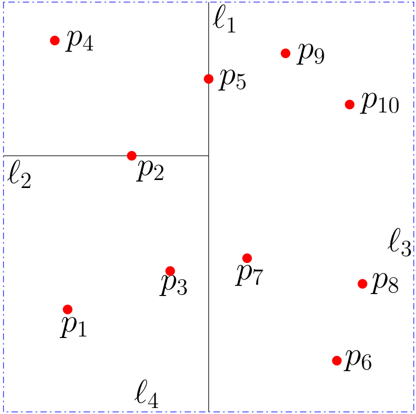


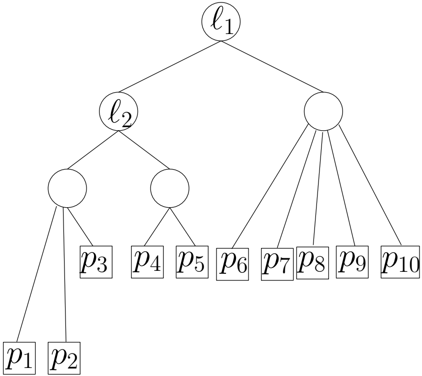

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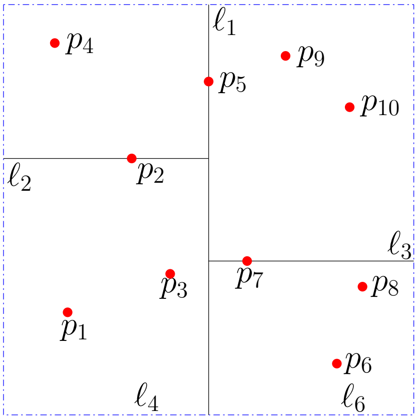


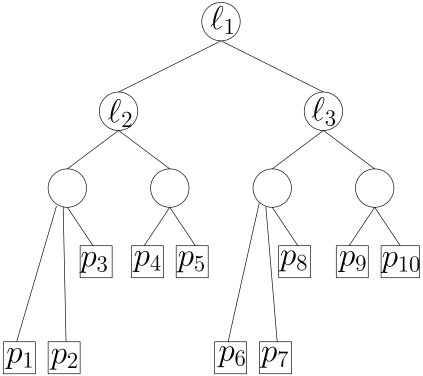

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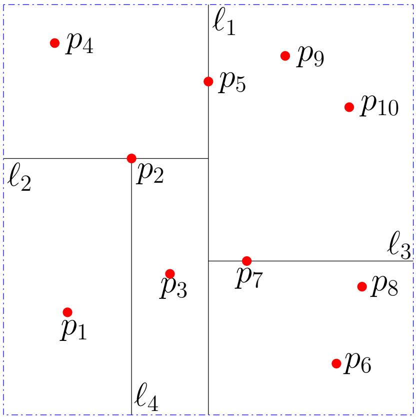


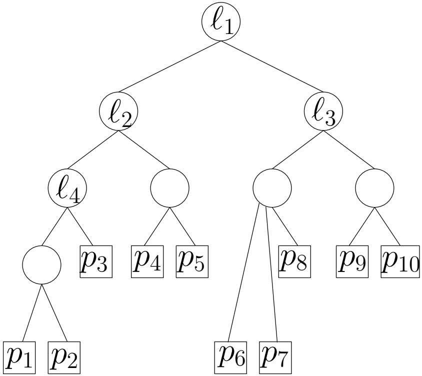

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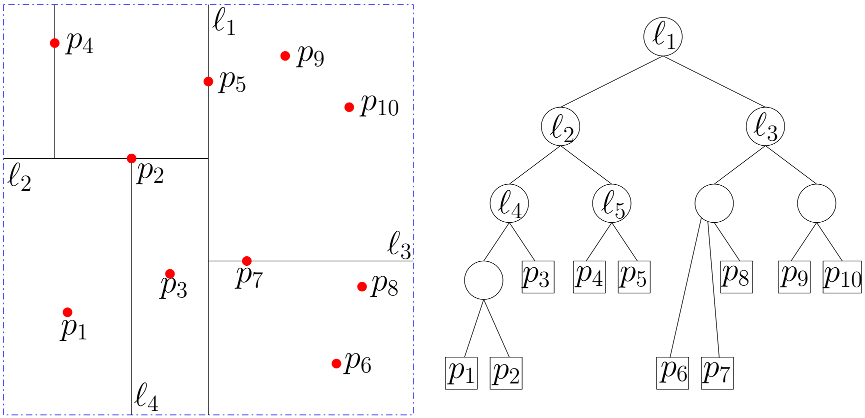

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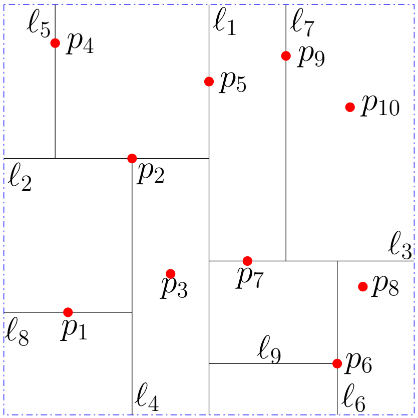
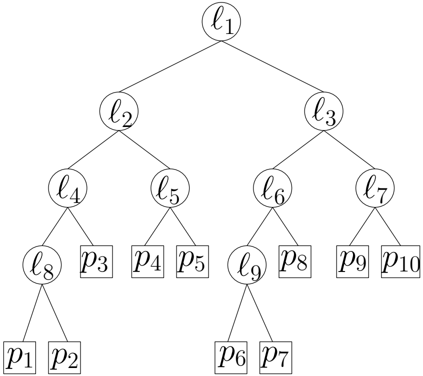


# Algorithm

**Algorithm 2** BuildKdTree($P$, *depth*)

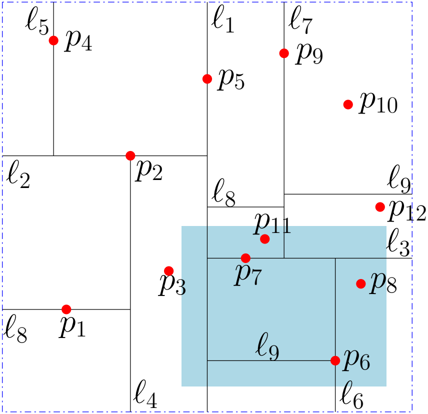
```
if P contains only one point then
    return a leaf storing this point
else if depth is even then
    Split P with a vertical line ℓ through the median x-coordinate into P1
    (left of ℓ) and P2 (right of ℓ)
else
    Split P with a horizontal line ℓ through the median x-coordinate into
    P1 (below ℓ) and P2 (above ℓ)
end if
left ← BuildKdTree(P1, depth + 1)
right ← BuildKdTree(P2, depth + 1)
Create a node v storing ℓ, make left left the left child of v, and make
right right the right child of v.
return(v)
```

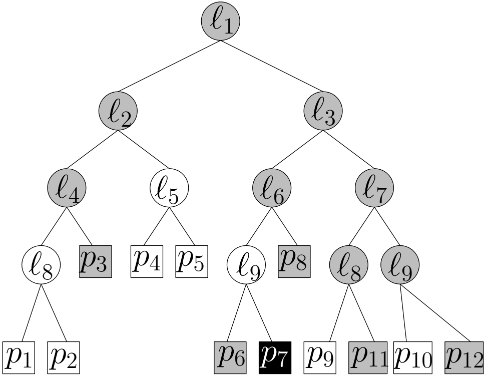
# Complexity

- The median of a set of $n$ values can be computed in $O(n)$ time
- Let $T(n)$ be the time needed to build a kd-tree on $n$ points

$T(1) = O(1)$
$T(n) = 2T(n/2) + O(n)$
A kd-tree can be built in $O(n \log n)$ time

# Kd-tree querying



White, grey, and black nodes with respect to region($v$):

- White node $v$: R does not intersect region($v$)
- Grey node $v$: R intersects region($v$), but region($v$) $\not\subseteq R$
- Black node $v$: region($v$)$\subseteq R$

# Complexity Analysis

- White node $v$: R does not intersect region($v$) Not visiting
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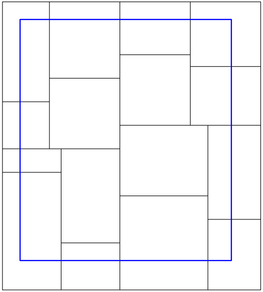

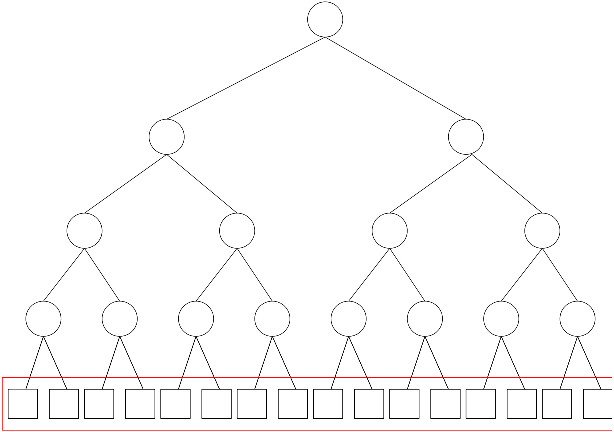

- How many grey nodes can be there among the leaf nodes.
- How many regions can be intersected by a axis parallel straight line.

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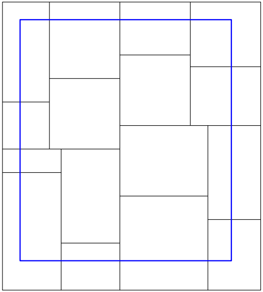

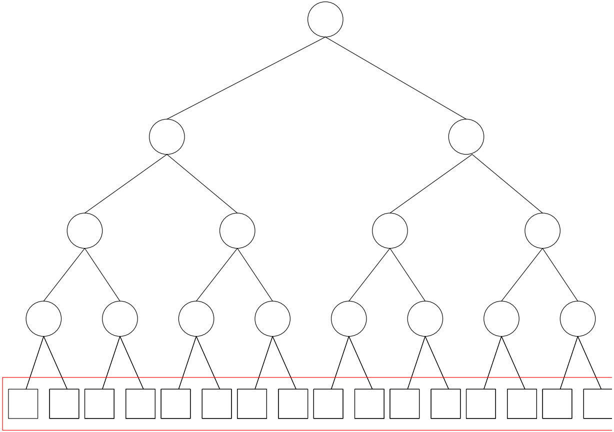

- At max $O(\sqrt(n))$
- In the previous level $O(\sqrt(n/2))$

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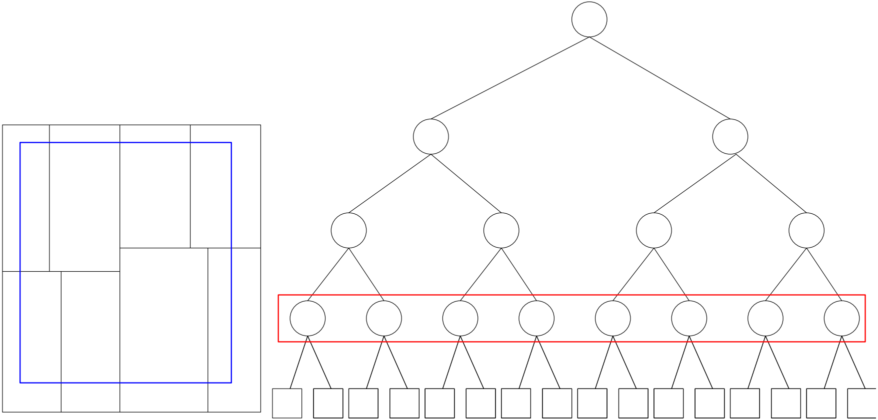

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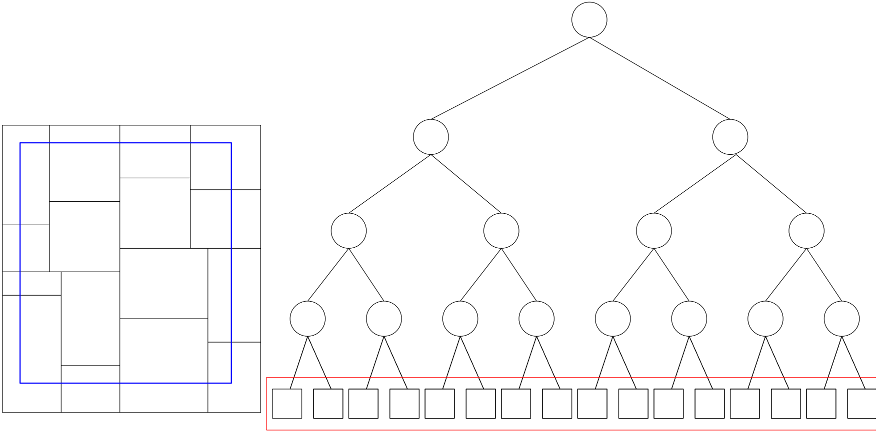

- Total no of Gray cells are $\sqrt{n}(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{8}} \ldots)$
- $O(\sqrt(n))$

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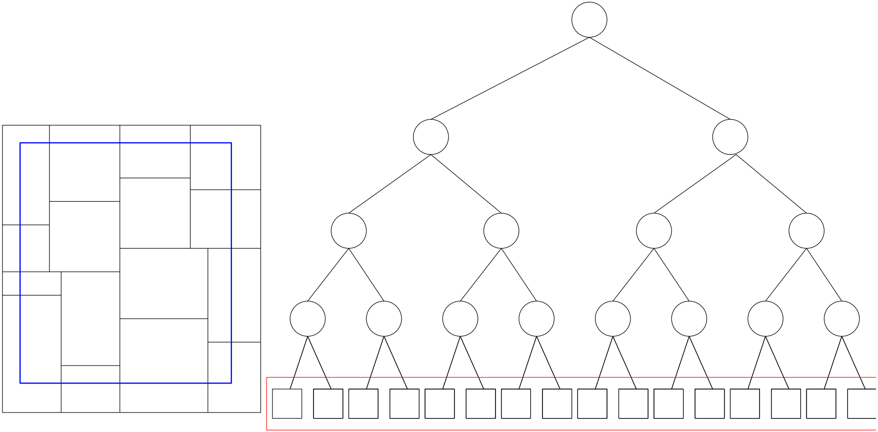

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# Higher dimensions

- A 3-dimensional kd-tree alternates splits on x, y, and z coordinate
- A 3D range query is performed with a box

**Theorem**

*A set of $n$ points in d-space can be preprocessed in $O(n \log n)$ time into a data structure of $O(n)$ size so that any d-dimensional range query can be answered in $O(n^{1-1/d} + k)$ time, where $k$ is the number of answers reported.*

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