Summer School on Graph Theory and Graph Algorithms

1. Design and implement an $O(nlogn)$ algorithm for computing the area of the union of n axis parallel rectangles.

2. Describe an $O(n)$ time algorithm to compute the intersection of convex polygons C_1 and C_2 where n is the cumulative number of vertices.

3. Given a set of points P on the plane, design an efficient algorithm to compute the diameter $D(P)$. The diameter of a P is defined as $max_{x,y \in P} dist(x, y)$.

   Hint:Prove that the pairs of points that determine the diameter are points on the convex hull. Modify the line sweep to an angular sweep method (also called the rotating callipers method).

4. Write a linear time algorithm for finding the convex hull of a simple polygon.

5. When we consider the Art Gallery Problem, we ask for the minimum number of guards that are sufficient to monitor a specific given polygon P. A set of guards is minimal if we cannot delete one of these guards without losing the complete coverage property (the set is minimal with respect to inclusion).

   (i) Give an example of a simple polygon P and a set of 5 guards that cover it such that deletion of any one guard causes part of the gallery P to be unseen (i.e., the set of 5 guards is minimal), but the guard number, $G(P)$, for P is less than 5: $(G(P) < 5)$.

   (ii) Give an example that the ratio between the number of guards in a minimal set and the number of guards in a minimum set cannot be bounded by a constant.

6. (Exterior point guards): Prove the following statement:$\lceil \frac{n+1}{3} \rceil$ point guards suffice to cover the exterior of any simple polygon with n vertices. Hint: $\lceil \frac{n+1}{3} \rceil = \lfloor \frac{n+3}{3} \rfloor$