1. Give an ILP formulation of minimum cost set cover problem and its corresponding LP relaxation. For any $e \in U$, let f_e be number of given subsets in which element e is present and $f = \max\{f_e : e \in U\}$. Obtain an f factor approximation algorithm for the minimum cost set cover problem, by a proper rounding of an optimal LP solution.

2. Consider an instance of the set cover problem, where $U = \{1, 2, \ldots, n\}$ and there are $n + 1$ subsets given by, $S_i = \{i\}$, with $c(S_i) = 1/i$, for $1 \leq i \leq n$ and and $S_{n+1} = U$ with $c(S_{n+1}) = 1.001$. What is the output of the greedy set cover algorithm on this instance? What is the cost of an optimum solution of minimum cost set cover for the given instance? What can you conclude about the worst case performance guarantee of the algorithm and its performance on the given instance?

3. A greedy algorithm for vertex cover is to keep on picking a vertex that covers the maximum number of uncovered edges, until all edges get covered. Give an instance of minimum vertex cover problem where greedy strategy will perform worse than 2 factor.

4. Give a tighter analysis of the greedy set cover algorithm and show that it achieves an approximation factor of H_k, where k is the cardinality of the largest specified subset of U.
   `Hint:` Consider an optimum set cover $F^* = \{S_1, S_2, \ldots, S_t\}$. For each S_i, upper bound price of covering elements of S_i using $H_k\ cost(S_i)$. This can be done by comparing cost of covering each element e_j of S_i using the greedy cover with the price of covering the same element using S_i itself. Order elements of S_i according to the order in which they were covered by the greedy algorithm.