Summer School on Graph Theory and Graph Algorithms

1. Let S be a set of n line segments in the plane. Let k denote the number of intersection points among line segments in S. Assume that for every intersection point there are just two line segments of S that pass through it, and there is no pair of line segments that have more than a single point in common. We are interested in computing a shortest horizontal line segment that intersects at least three line segments of S and is as short as possible. Write down a plane sweep algorithm to solve this problem.
2. Is it possible to construct a point set with three sites whose Voronoi vertex is exterior to the triangle determined by the sites?
3. Prove that there is no algorithm that can compute the Voronoi diagram of n sites faster than $nlogn$ in the worst case
4. The edges of both the Delaunay and Voronoi diagrams are line segments. Give a simple necessary and sufficient condition on a pair of sites A and B so that AB is a Delaunay edge and AB intersects its dual Voronoi edge.
5. Any set of points in the plane implicitly defines a weighted, complete graph whose vertices are the points themselves, whose edges are line segments, and where the weight of an edge is its Euclidean length. A Euclidean minimum spanning tree of a set of points is a minimum spanning tree of this graph.

   Prove that Euclidean minimum spanning tree is a subgraph of Delaunay triangulation.