

Feedback Vertex Set Problem

Input $G(V, E)$

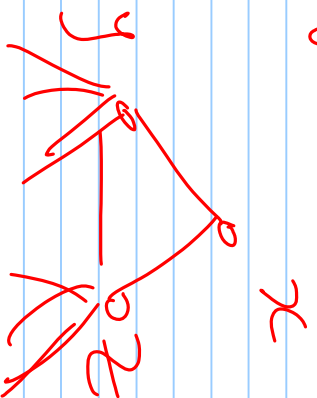
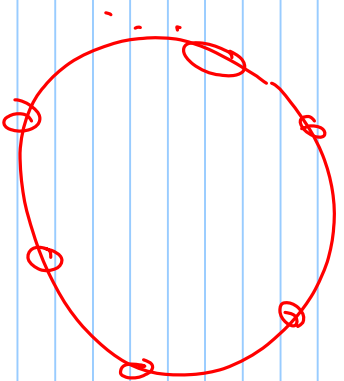
Parameter k

Qn: $\exists S \subseteq V, |S| \leq k, \text{ s.t. } G-S \text{ is a forest}$

(Exercise: $O(n)$ algm to detect

whether G has a cycle).

Try all subsets of size $\leq k \sim O(n^{k+c})$



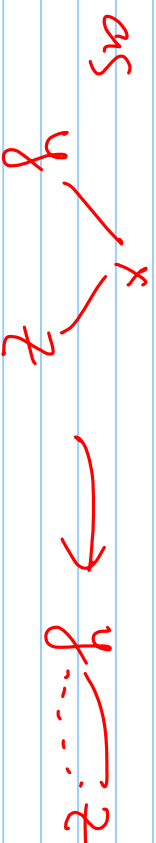
Rule 1: delete vertices with

degree ≤ 1

Rule 2: If x is of degree

2 with y and z as

neighbors, then "Short circuit"



Rule 3: If x has a self loop, include

X in Soln, $k \leftarrow k-1$

Resulting graph (G, k)

$\text{Mindegree}(G) \geq 3$

Thm: Any n -vertex graph has a cycle of length $\leq 2 \log n + 1$

and can be found in $O(m \log n)$ time.

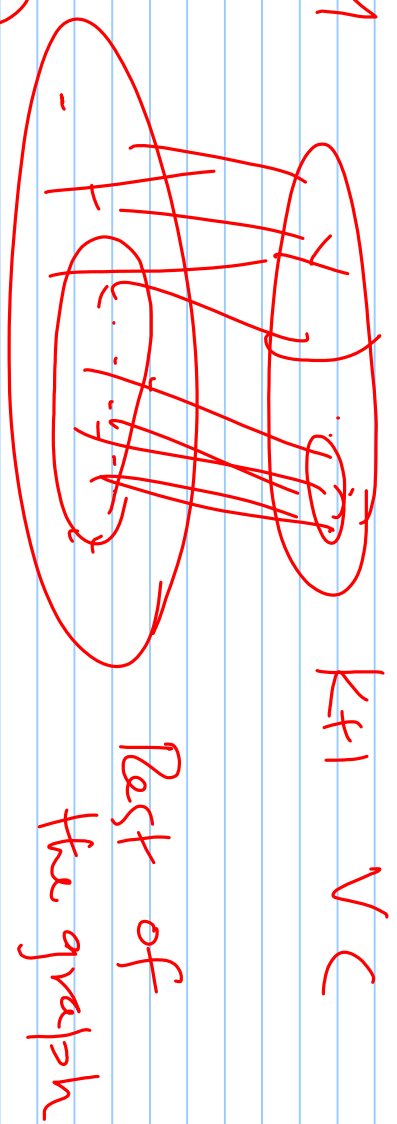
$O((\log n)^k (m+n))$

Iterative Compression

(G, k) along with a soln of size $k+1$

Qn: \exists a soln of size $\leq k$? M

Try all subsets Y of M
 and $G[M-Y]$ is indep
 and $\{U(M-Y) \cup (V-M)\}$ in the
 soln.
 $S \leftarrow Y \cup (V-M)$

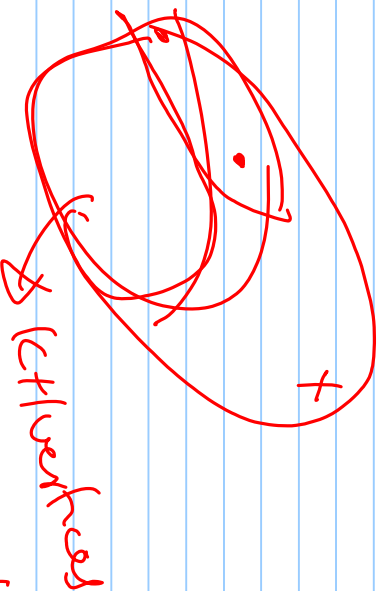


- Finding size among solns for all S .
- If its size $\leq k$, return Yes, else return No.

$$O^*(2^k)$$

How do we get a soln of size $k+1$?

G_{k+2} - subgraph on vertices $1, 2, \dots, k+2$



$\exists ? k$ sized soln

yes /

no

return from stop

Iterate by including $k+3$ in graph & vertex cover

and continue.

Overall $O(2^k) \sim O(2^k n)$

Iterative Compression for EVS.

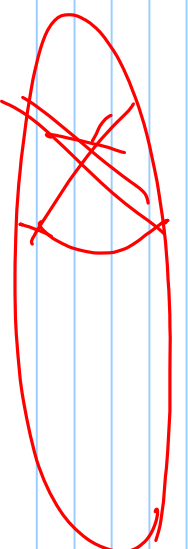
Compression Step:

Input: $G(V, E)$, k

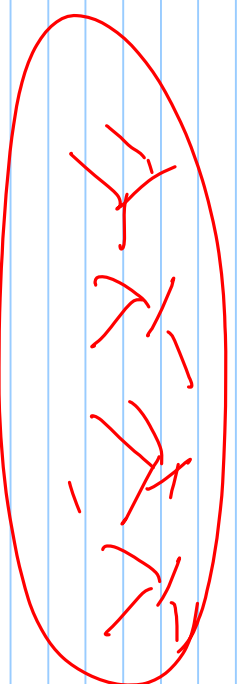
M - EVS of size $k+1$

Suppose Y is SNM

Soln looking for



$(M) = k+1$



$V-M$
forest

Rule,

Suppose

$x \in V - M$

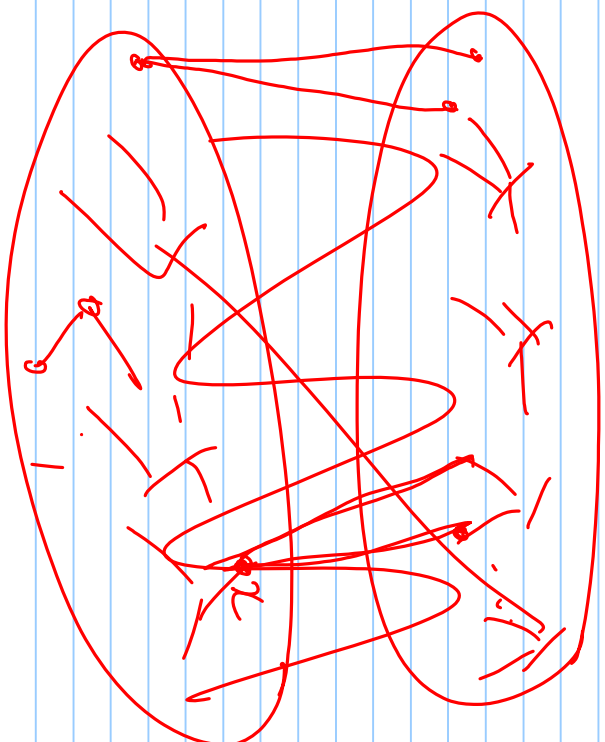
has ≥ 2 nbrs

in one component

of $M - Y$,

pick x : into soln

$k \leftarrow k-1$

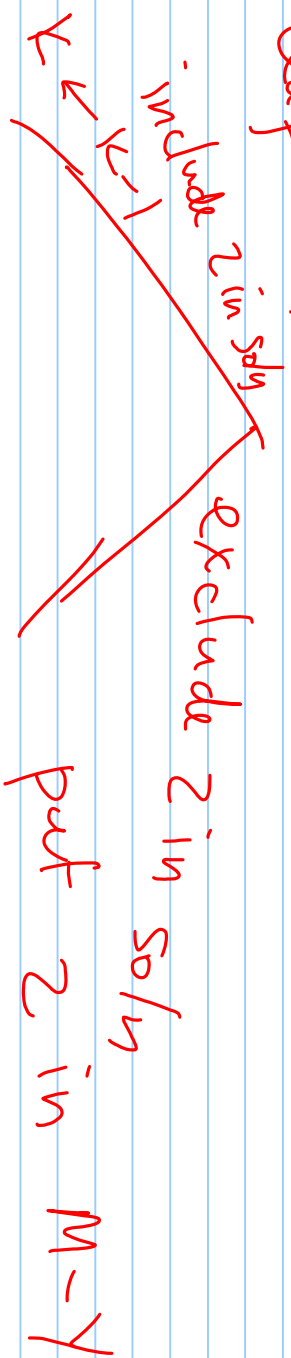


$$|M - Y| = |k + 1 - Y|$$

$V - Y$

Branching Rule:

Pick a leaf vertex z of a tree in $V-M$



z has at least 2 nbrs in $M-Y$ and so # components in $M-Y$ drops by at least one.

So $M = K + \# \text{ Comps in } M-Y$
 $\leq |K-Y|$

M drops by at least 1 in each branch

So the runtime is $O^*(2^k)$ which is $O^*(2^{2k-1})$

is $O^*(4^k)$ for a fixed Y .

and hence $O^*(8^k)$ overall.

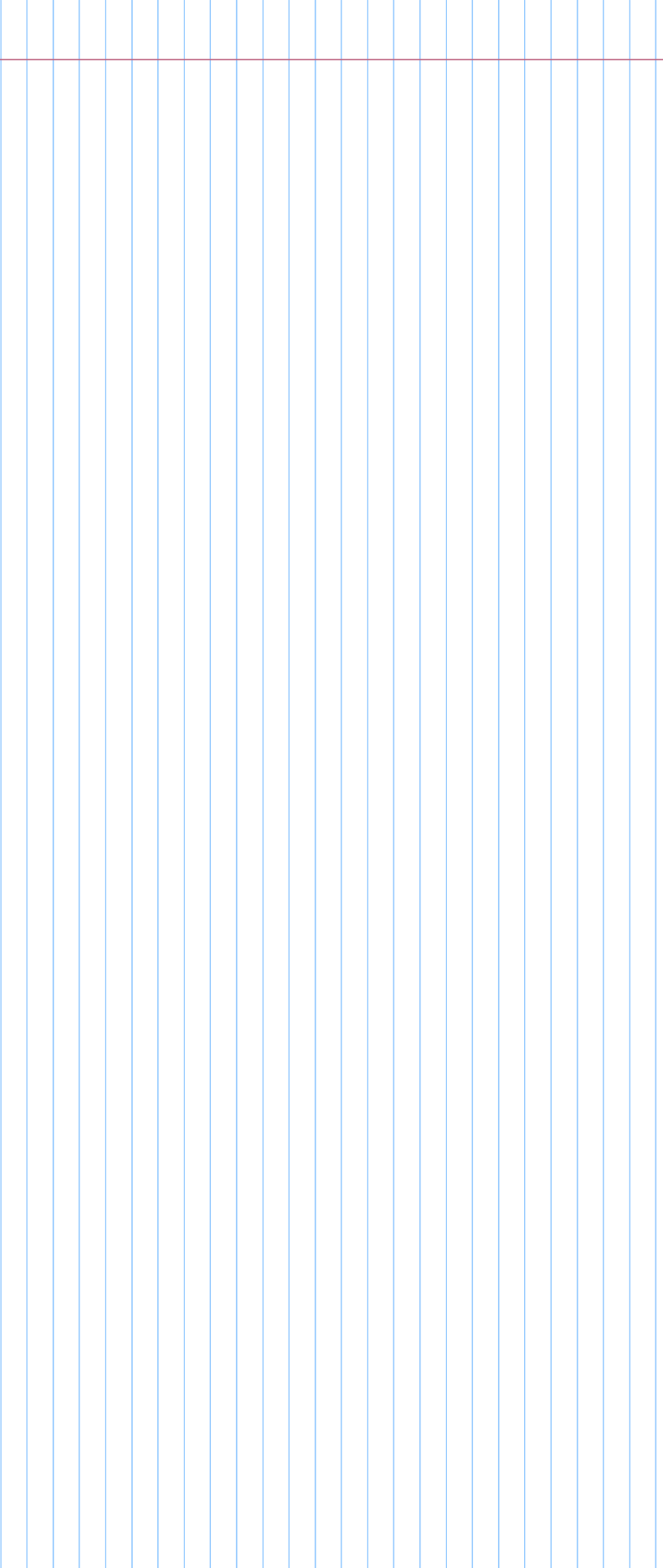
A more careful analysis gives $O^*(5^k)$

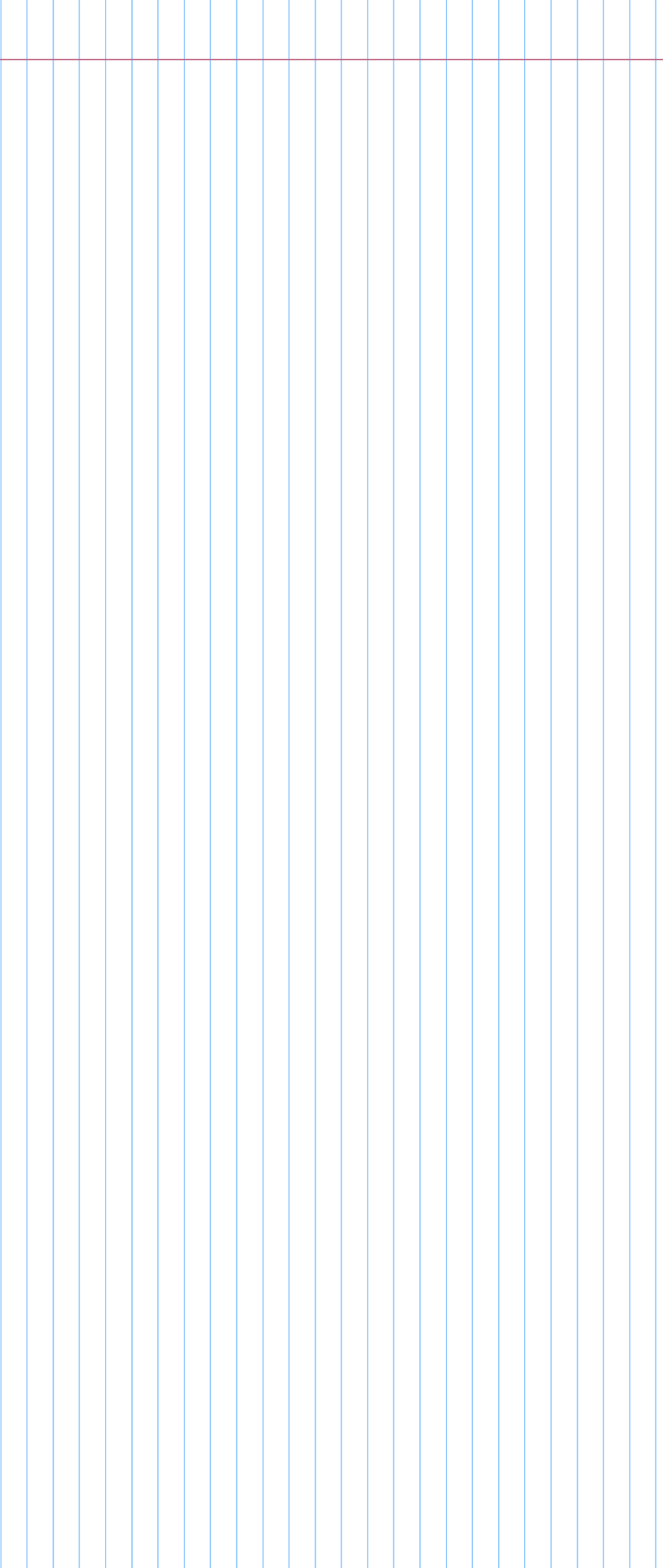
For VC, the current best is $O^*(1.28^k)$

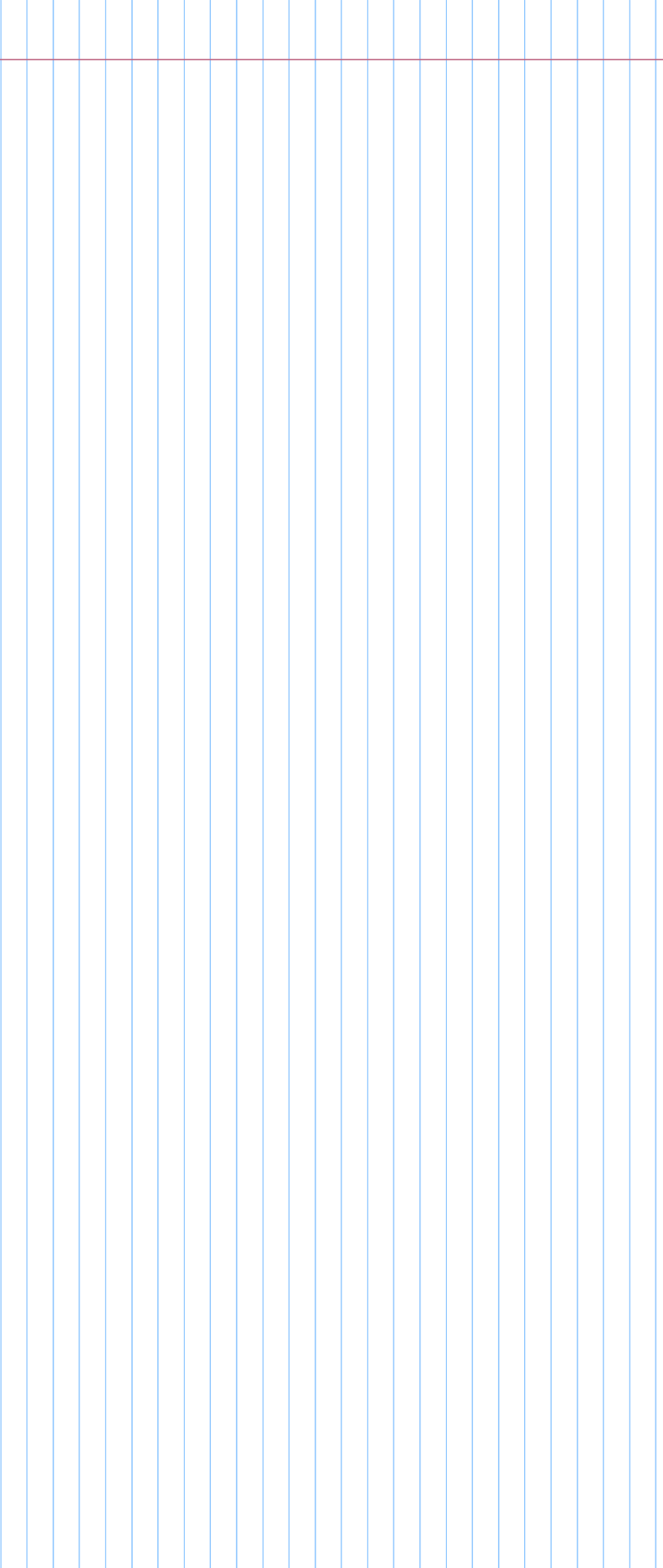


Area with vertical blue lines for writing.

Vertical rectangular box with a dotted line on the left side, likely for a page number or label.









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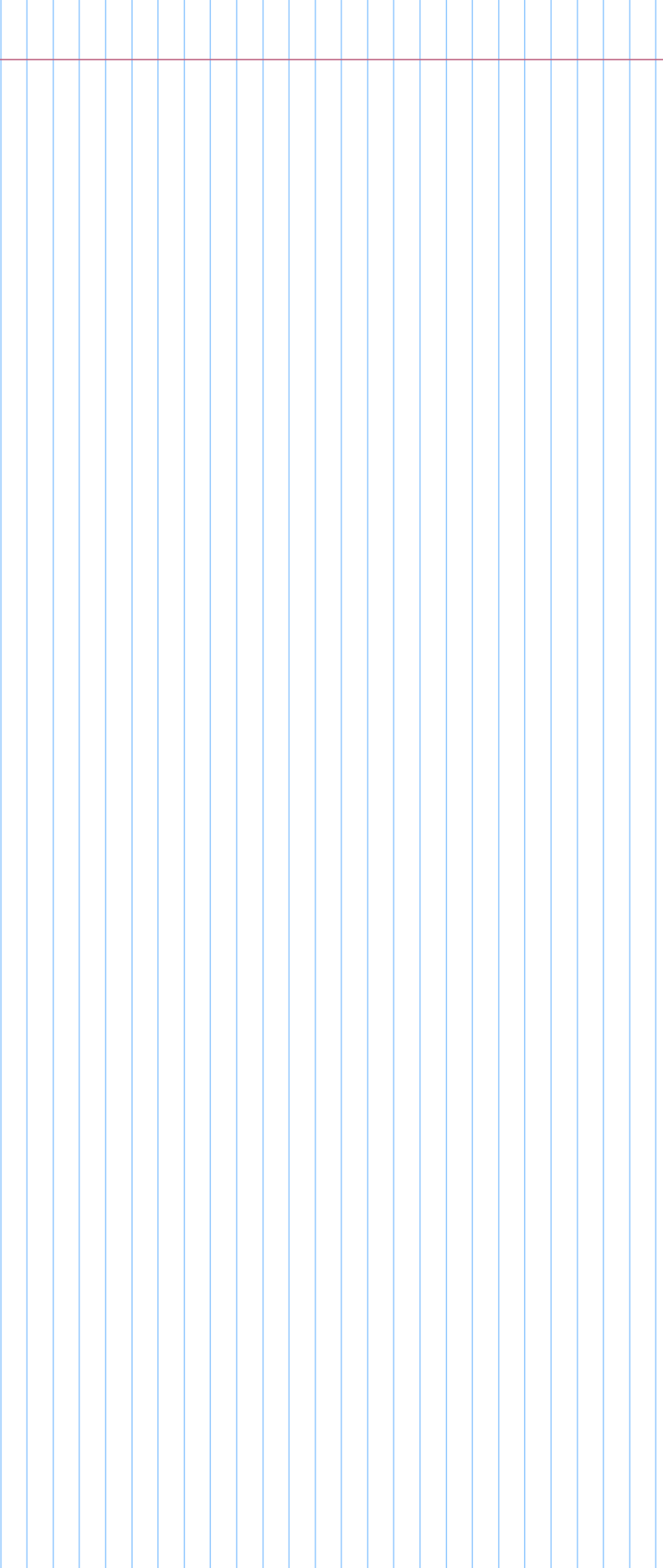
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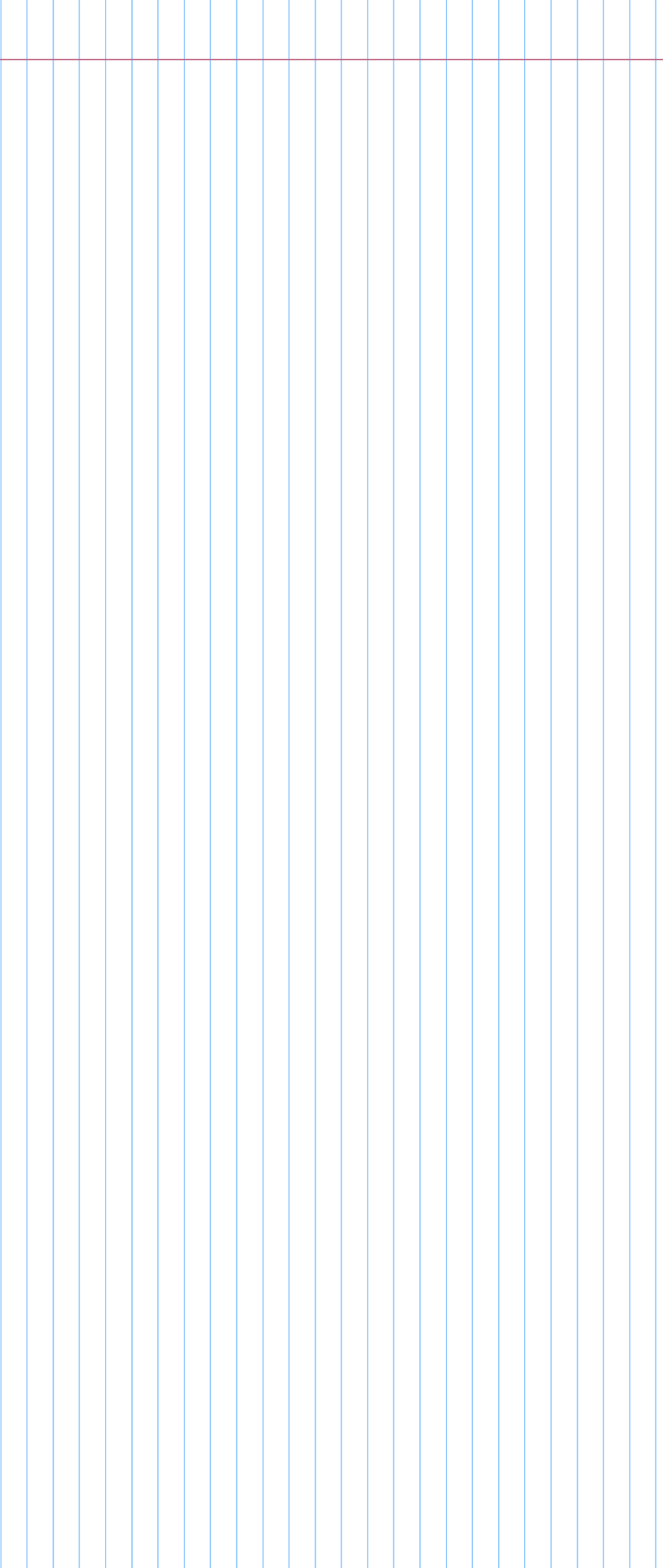
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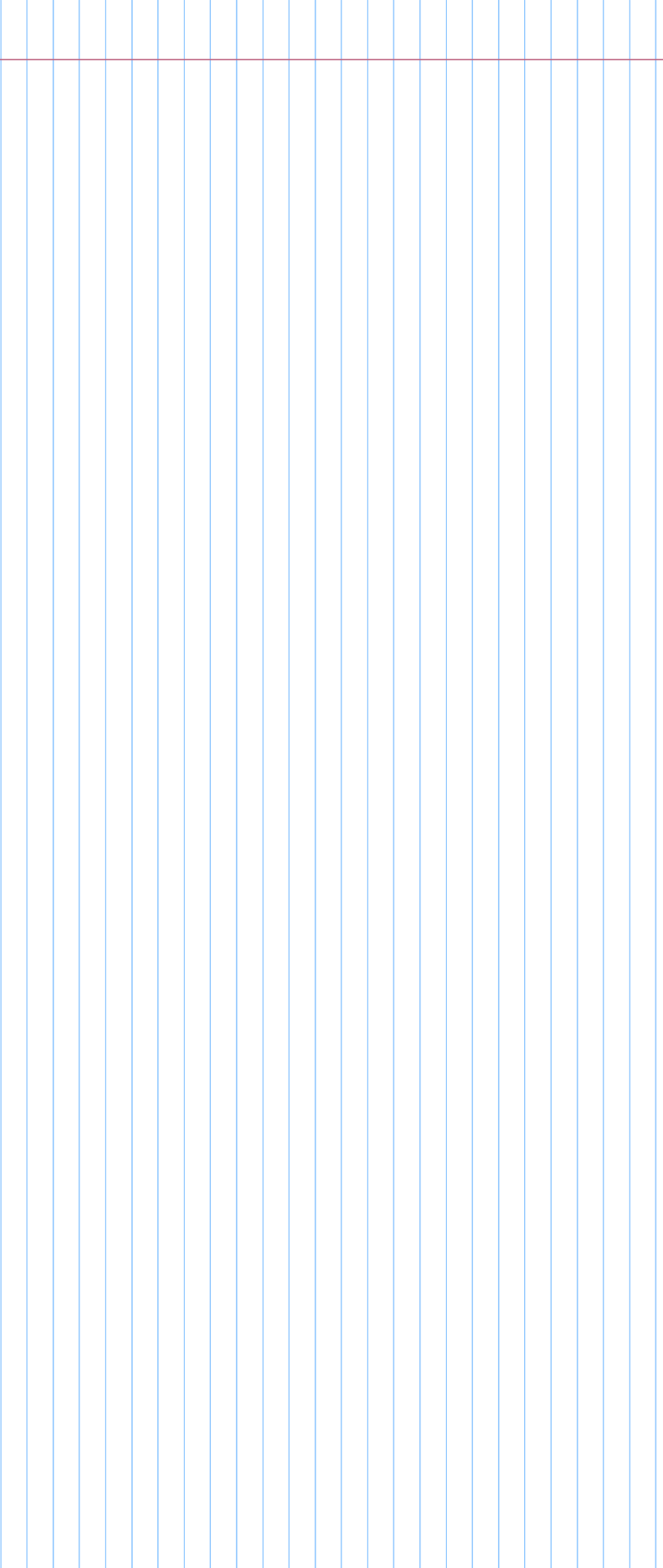
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