A problem Π is NP-complete

(Vertex Cover, Hamiltonian Cycle)

Unless $P = NP$

Π has no algm running in time poly in input size

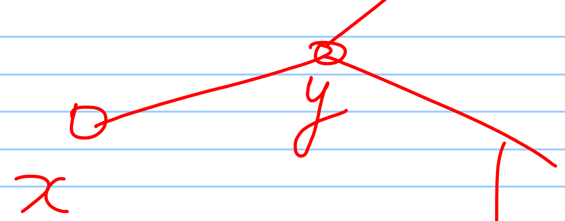
that gives an exact answer

for all possible inputs

Vertex Cover on Trees

1. Delete isolated vertices

2. x is of degree 1 with y as nbr, pick y into the soln.

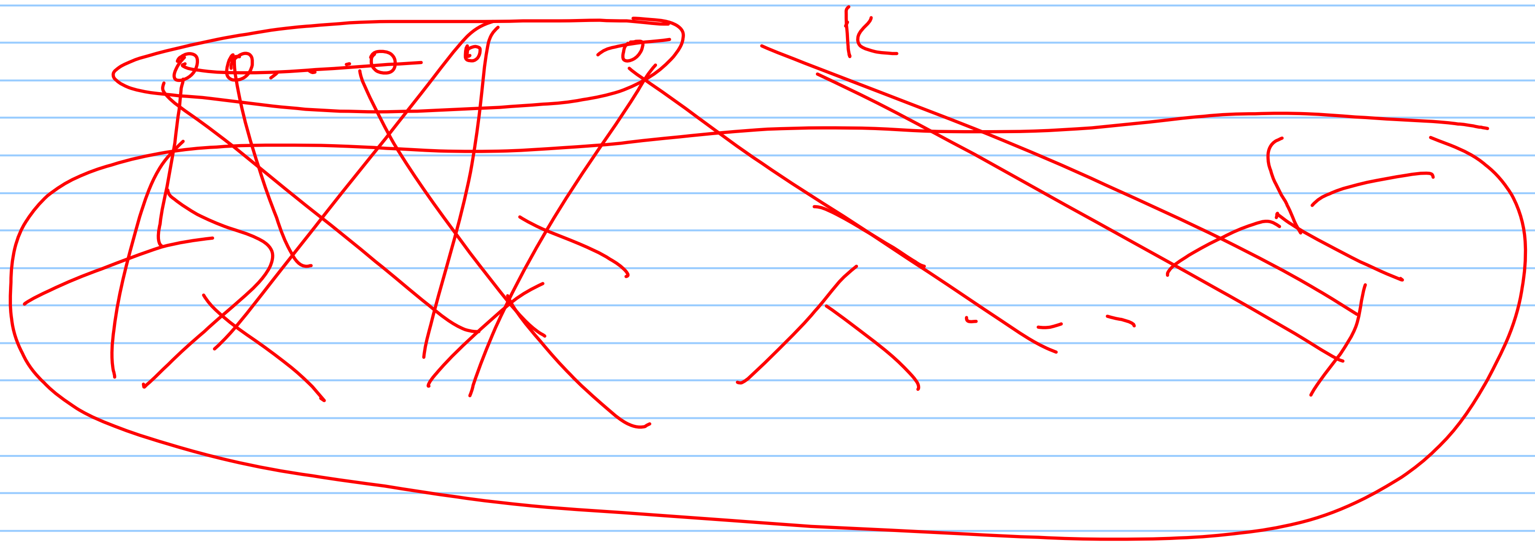


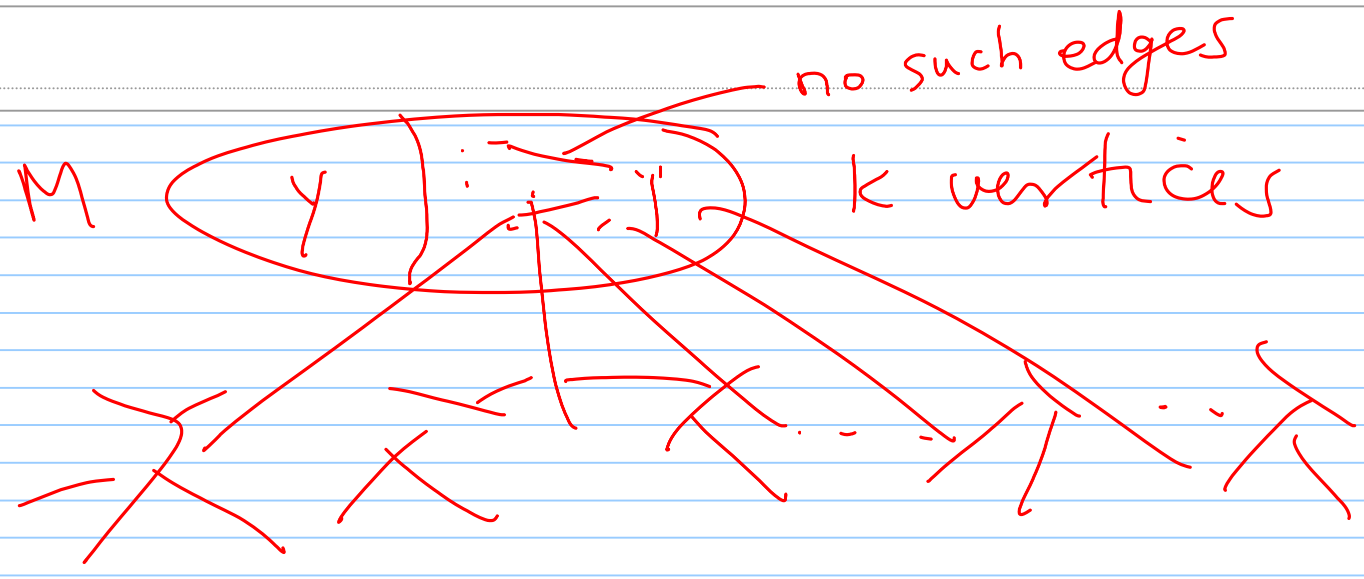
(Exercise: Show that picking high degree vertices don't necessarily give optimum)

3. Delete y, and continue (recurse)

G is a ~~tree~~ forest T + k vertices

forest





Suppose the soln S intersects M in some set Y.

$N((M-Y)) \cap (V-M)$ must be in S.

Remaining vertices form a forest.

For every $Y \subseteq M$,

- let S be a min VC of $G[V - M - N(M-Y) \cap (V-M)]$
- $S' = S \cup Y \cup N(M-Y) \cap (V-M)$

Pick the S' of smallest size.

$O(2^k n^c)$

$O^*(2^k)$

Input — x, k ← parameter

fixed Parameter Tractable (fpt)

if $O(f(k)\,|x|^{c})$ ← independent of k

($|x|$ ← n)

$O^{*}(f(k))$ algm.

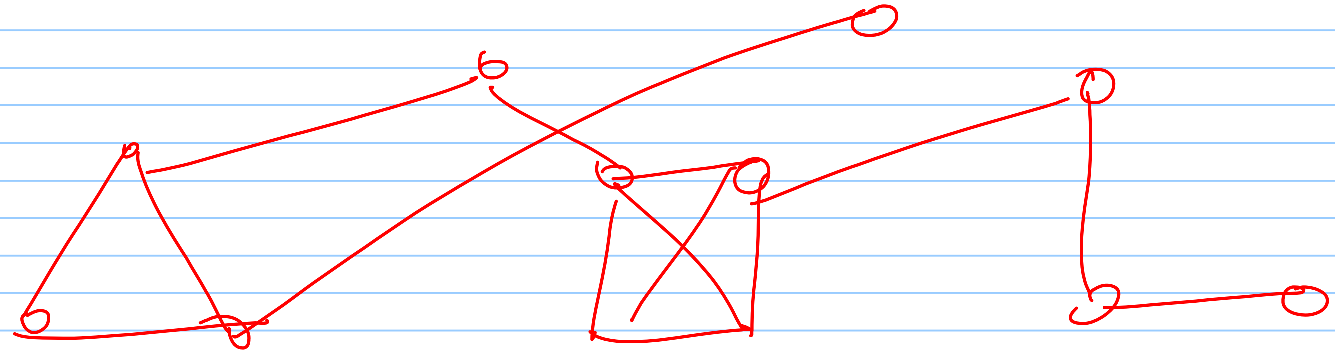
Cluster Graph.

every connected component is a clique.

Problem: Given $G = (V, E)$, k,

does $\exists\, S \subseteq V$, $|S| \leq k$ such that

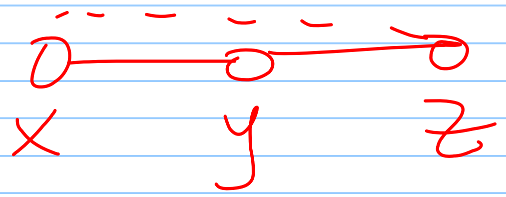
$G - S$ is a cluster graph?



ii) Try all $\leq k$ sized subsets to delete and check $\sim O(n^{k+2})$

Is it fpt? Goal: $O(f(k)\, n^3)$

G is a cluster graph if and only if G has no induced path on 3 vertices



$(x, y) \in E$

$(y, z) \in E$

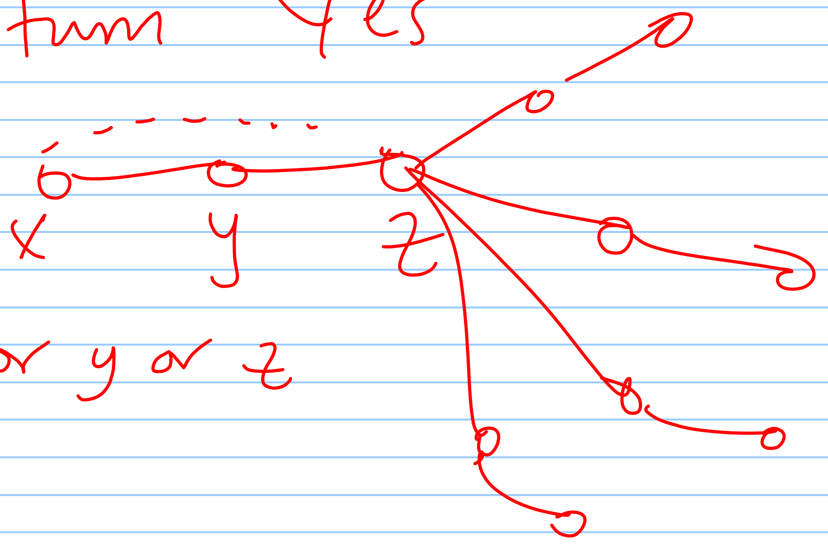
$(x, z) \notin E$

F.P.T algorithm (G, k) $k \geq 0$

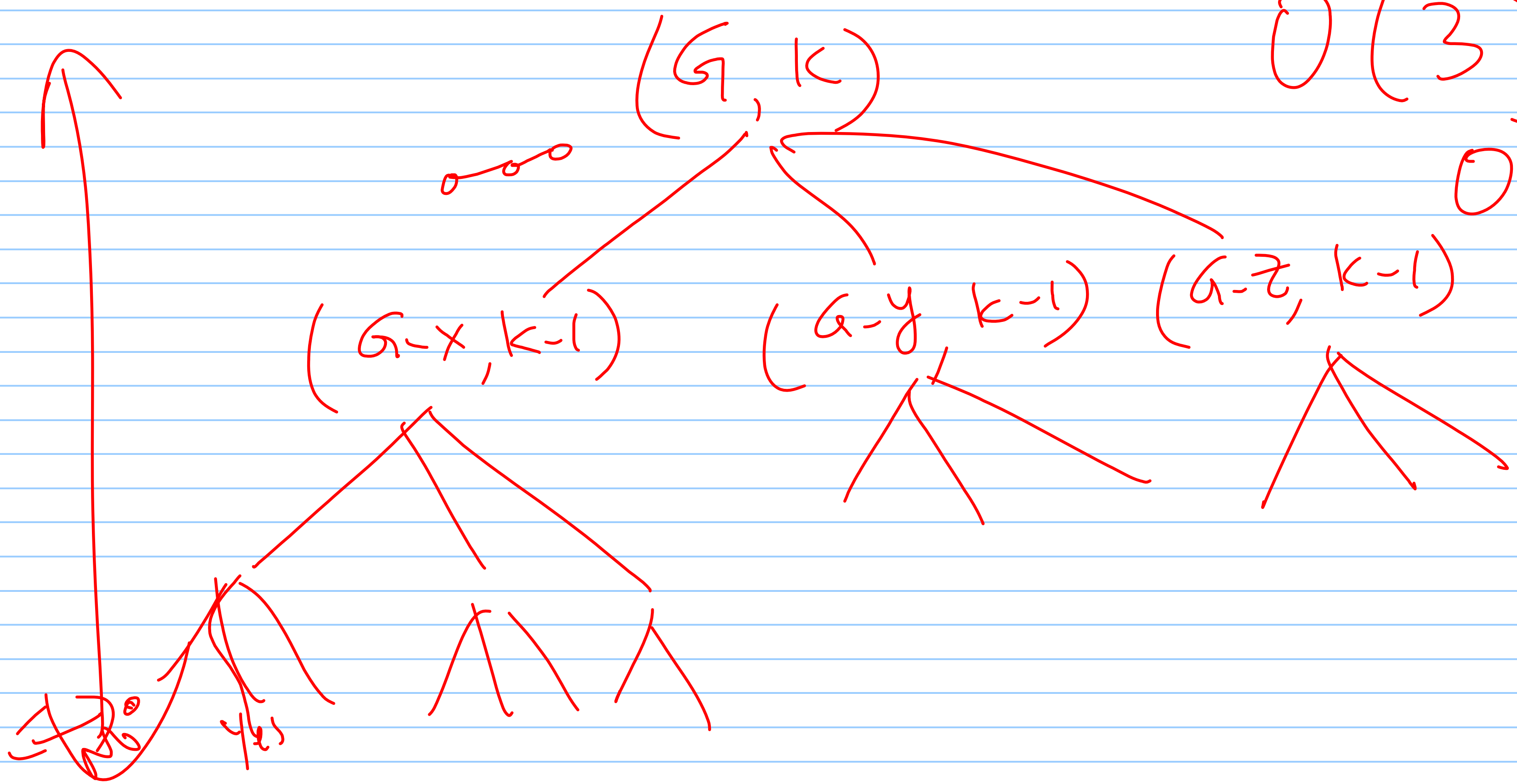
1. check if G is a cluster graph

   if yes then return yes

   else find a P_3



2. Branch by deleting x or y or z



$O(3^k n^2)$

$O^*(3^k)$