

Input -  $G, K$

Kernelization/Preprocessing

$(G, K) \xrightarrow{\text{Polynomial time}}$

$(G', K')$   
 $|G'|, K'$  are just

$|E| = m$   
 $k$ -parameter

functions of  $k$   
 $\phi$  kernel

$$f(k) n^c$$

Input:  $G(V, E)$ ,  $|V| = n$ ,  $|E| = m$

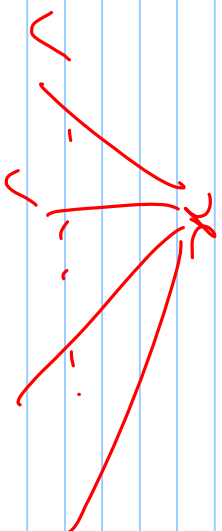
$k$ : parameter

Qn: Is  $SSV$ ,  $|S| \leq k$ ,  $G-S$  is edgeless?

Rule 1: Delete isolated vertices

Rule 2: If  $x$  is of degree 1 with  $y$  as its unique nbr, include  $y$  in soln, delete  $y$   
 $k \leftarrow k-1$ .

Rule 3: If  $x$  is of degree  $> k$ ,



include  $x$  in Soln

$k \leftarrow k-1$

delete  $x$

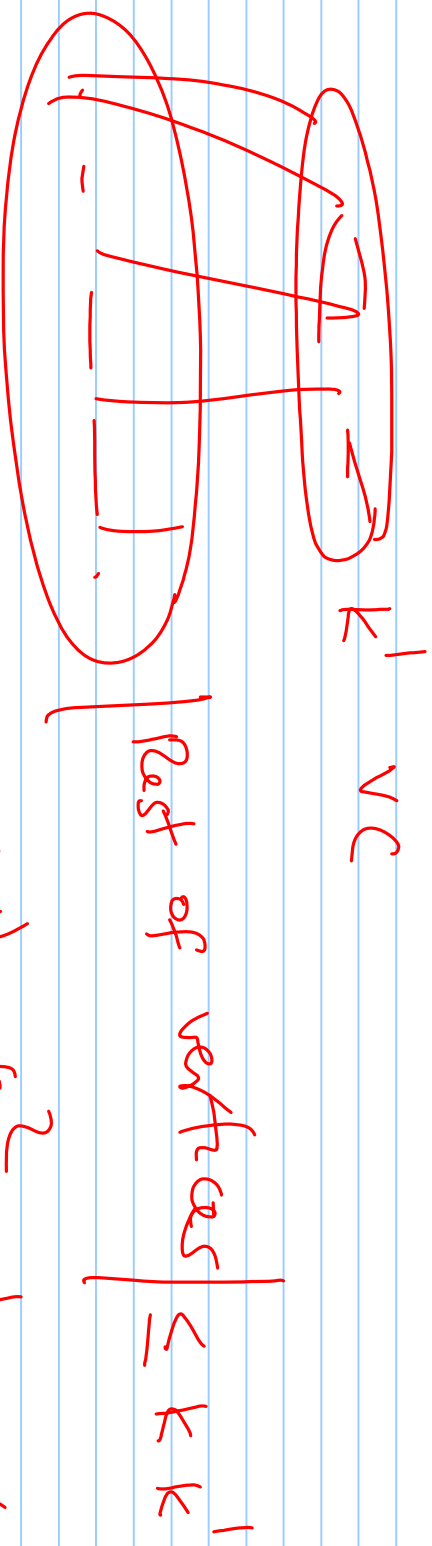
Resulting graph  $(G', k')$

$G'$  has

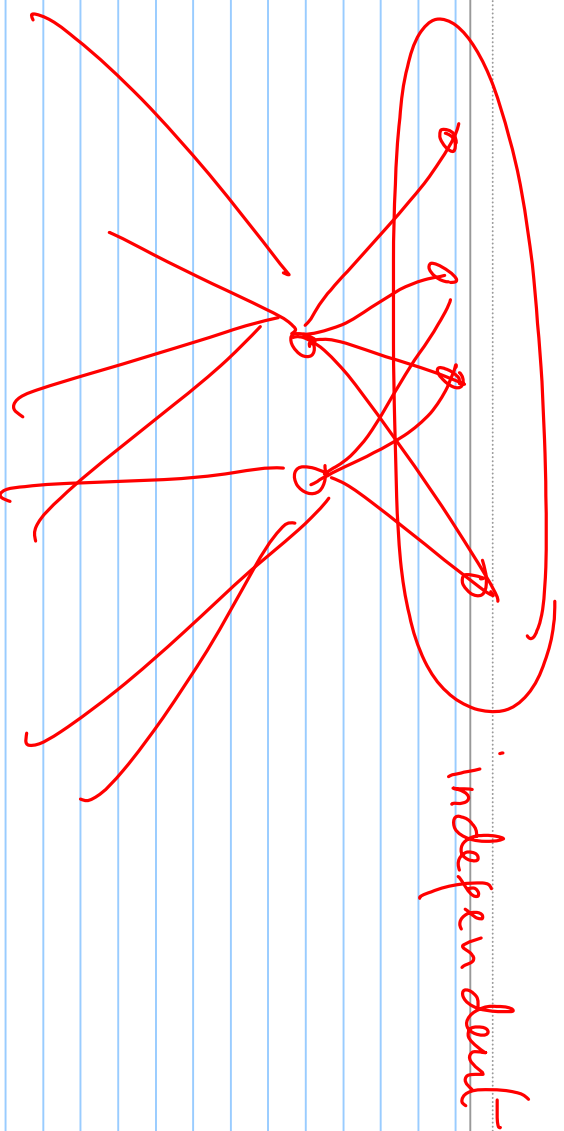
$\left\{ \begin{array}{l} \text{no isolated vertices} \\ \text{no deg 1 vertex} \\ \text{degree} \leq k \end{array} \right.$

Claim: Suppose  $G'$  has a  $k'$  sized VC, then

$$|V(G')| \leq (k+1)k', \quad |E(G')| \leq k'k$$

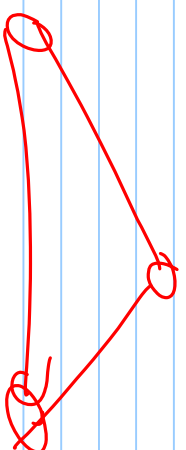


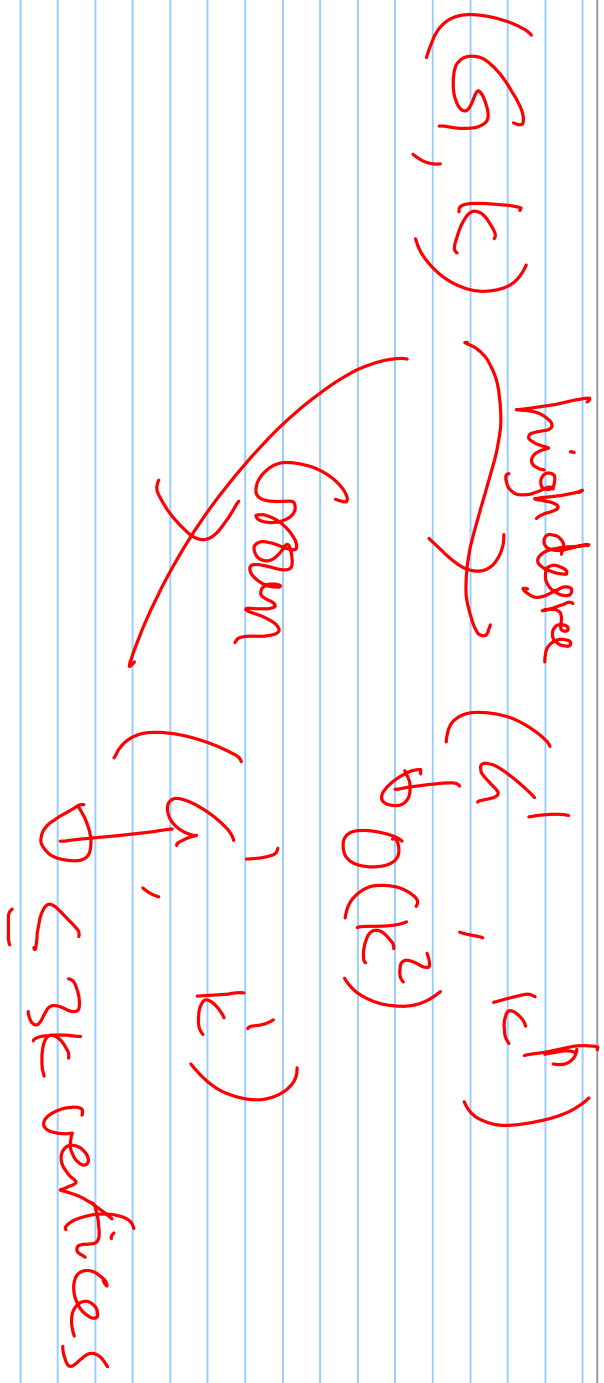
Rule 4: If  $|V(G')| > k+k$  or  $|E(G')| > k^2$ , return NO.



$M$  - matching  
 $S$  - vertex cover

$$|M| \leq |S|$$





Input -  $G(u, E)$

Minimize  $\sum_{v \in V} x_v$

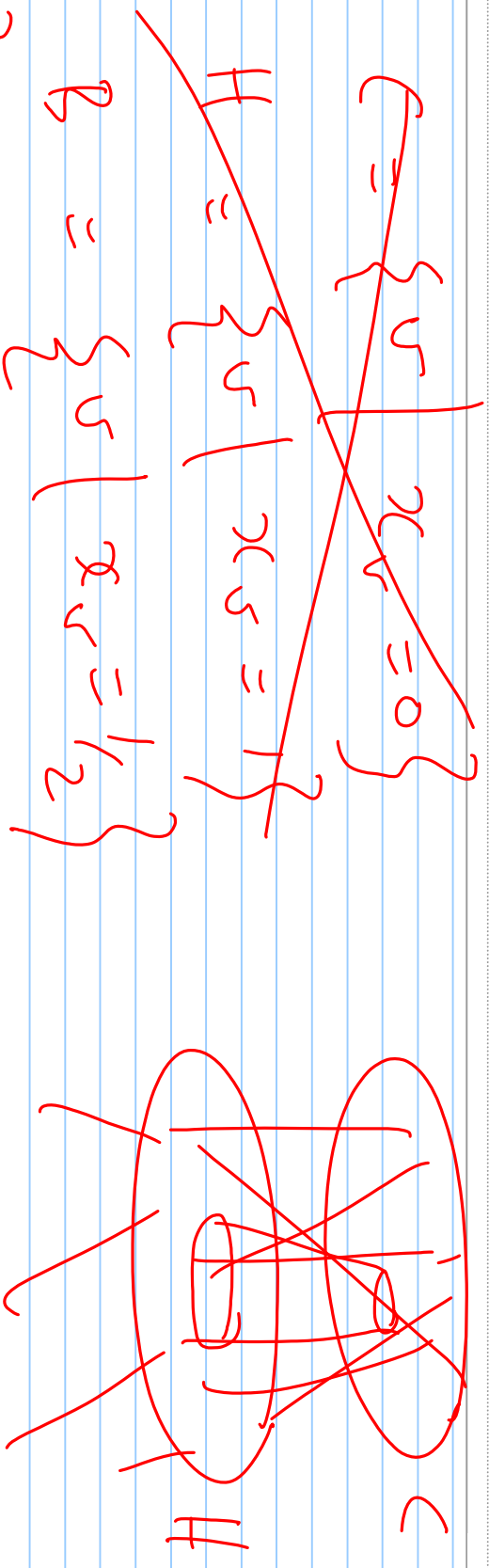
s.t.

$$x_u + x_v \geq 1 \quad \forall (u, v) \in E$$

$$x_u \geq 0$$

Thm 1:  $\exists \{0, \frac{1}{2}, 1\}$  opt soln for this LP.

Thm 2:  $\exists$  integer VC soln that contains all of 1 values and none of 0 values.



$\{C, H, B\}$  form a cover.

To prove:  $\exists$  matching saturating  $H$  between

$C$  and  $H$ . We'll prove  $V \cap S \subseteq H$   
 $|N(S) \cap C| \geq |S|$



$$LP_{opt} = \sum_{v \in V} x_v^*$$

$$|MinVc| \geq LP_{opt}$$

If  $LP_{opt} > k$  then return NO

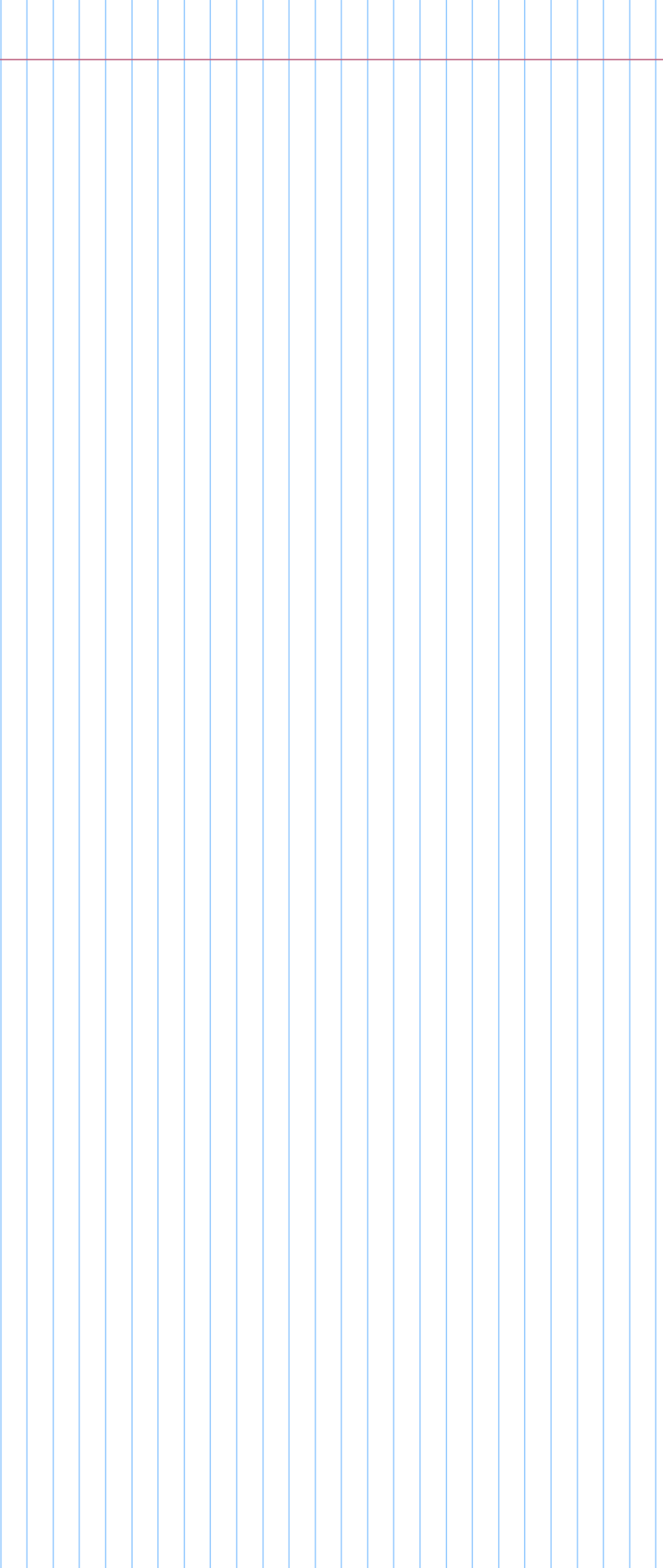
$$LP_{opt} \leq k$$

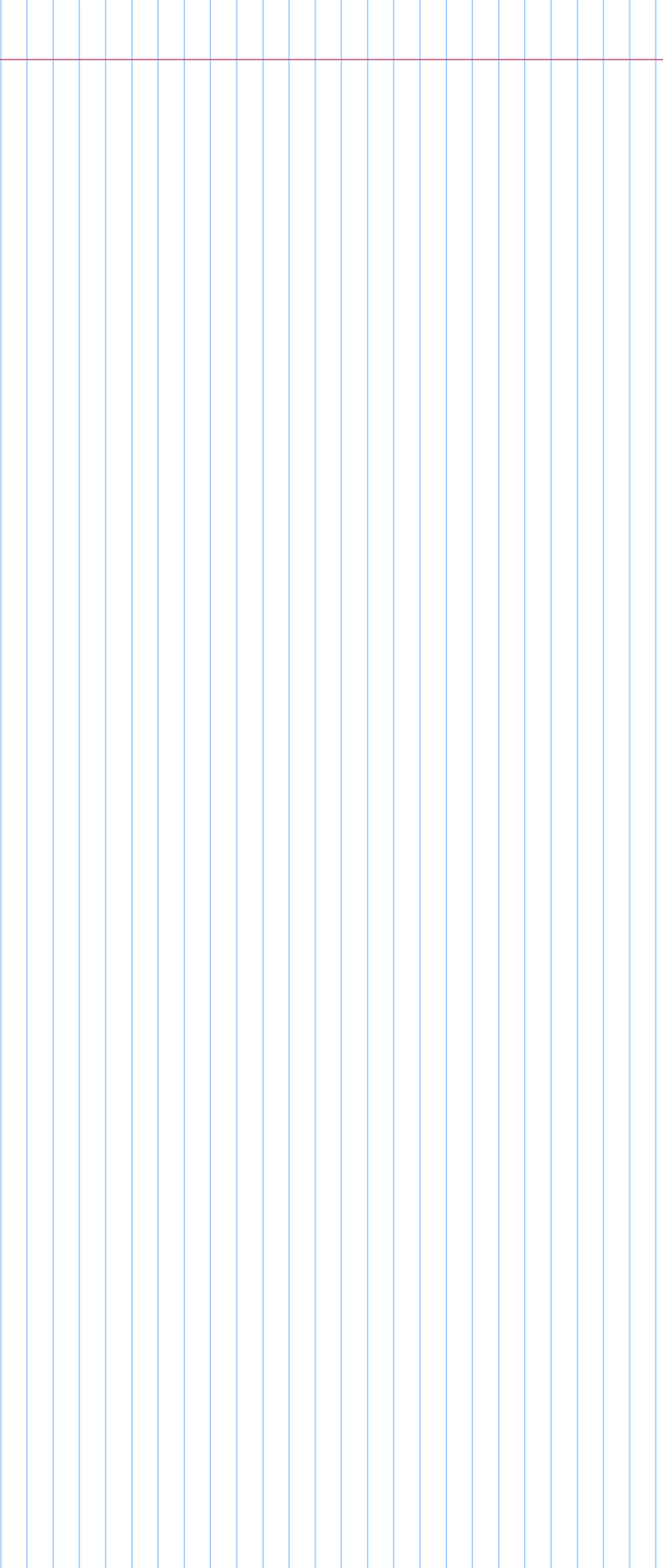
$\Rightarrow |B| \leq 2k$  and we have a  $2k$  vertex kernel for vertex cover.

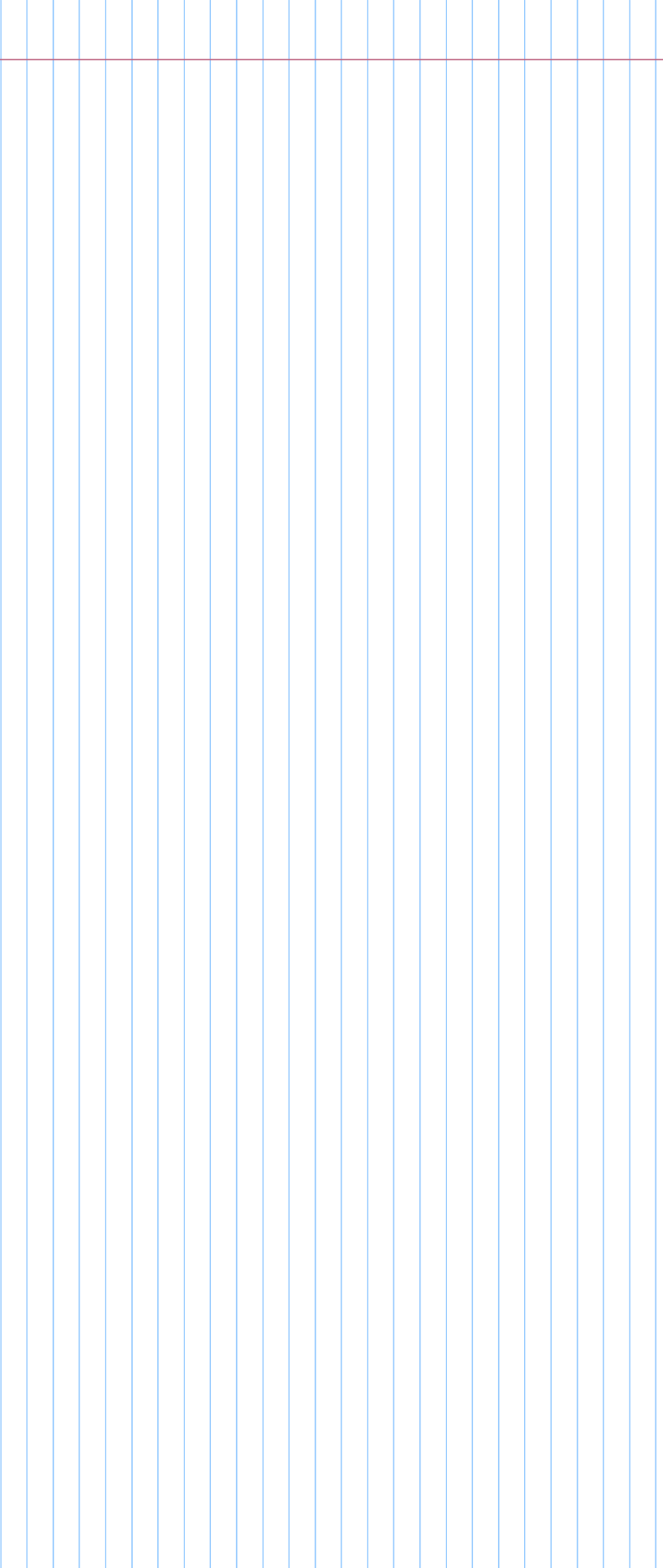


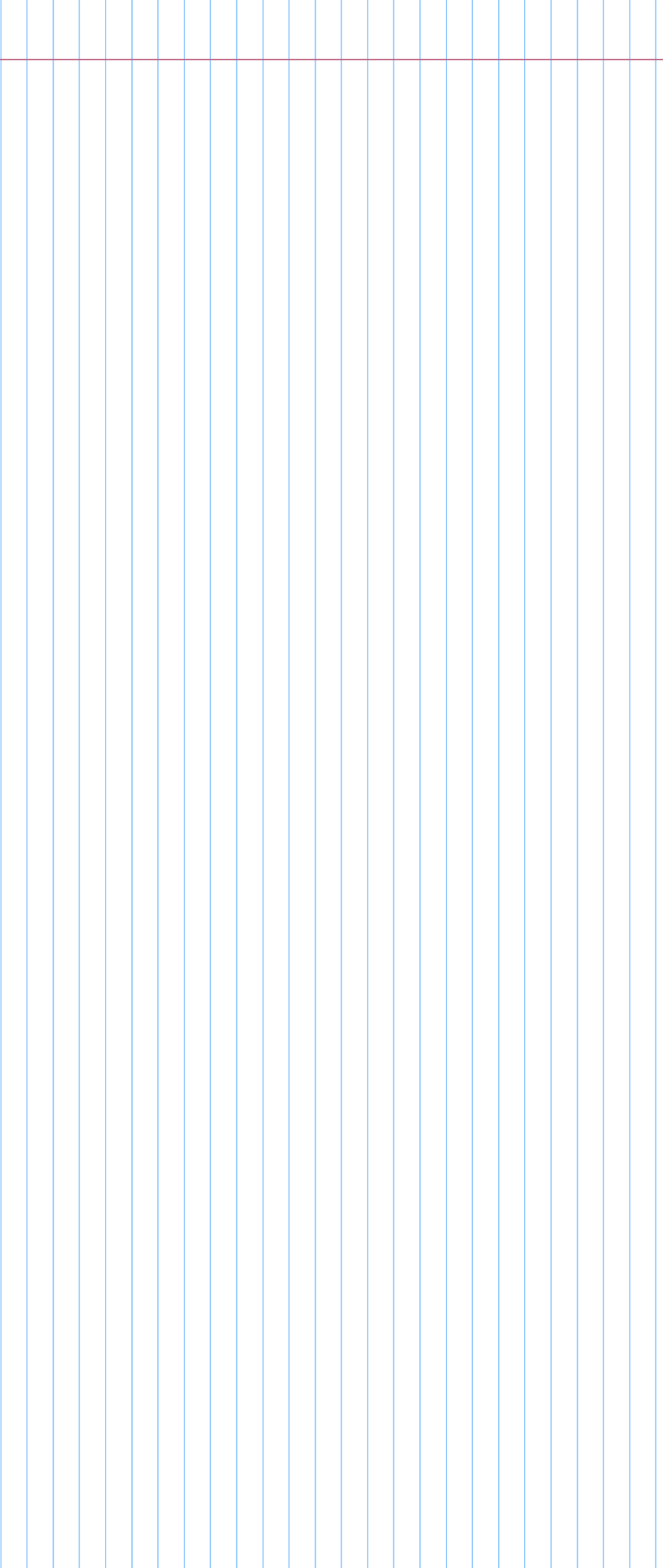
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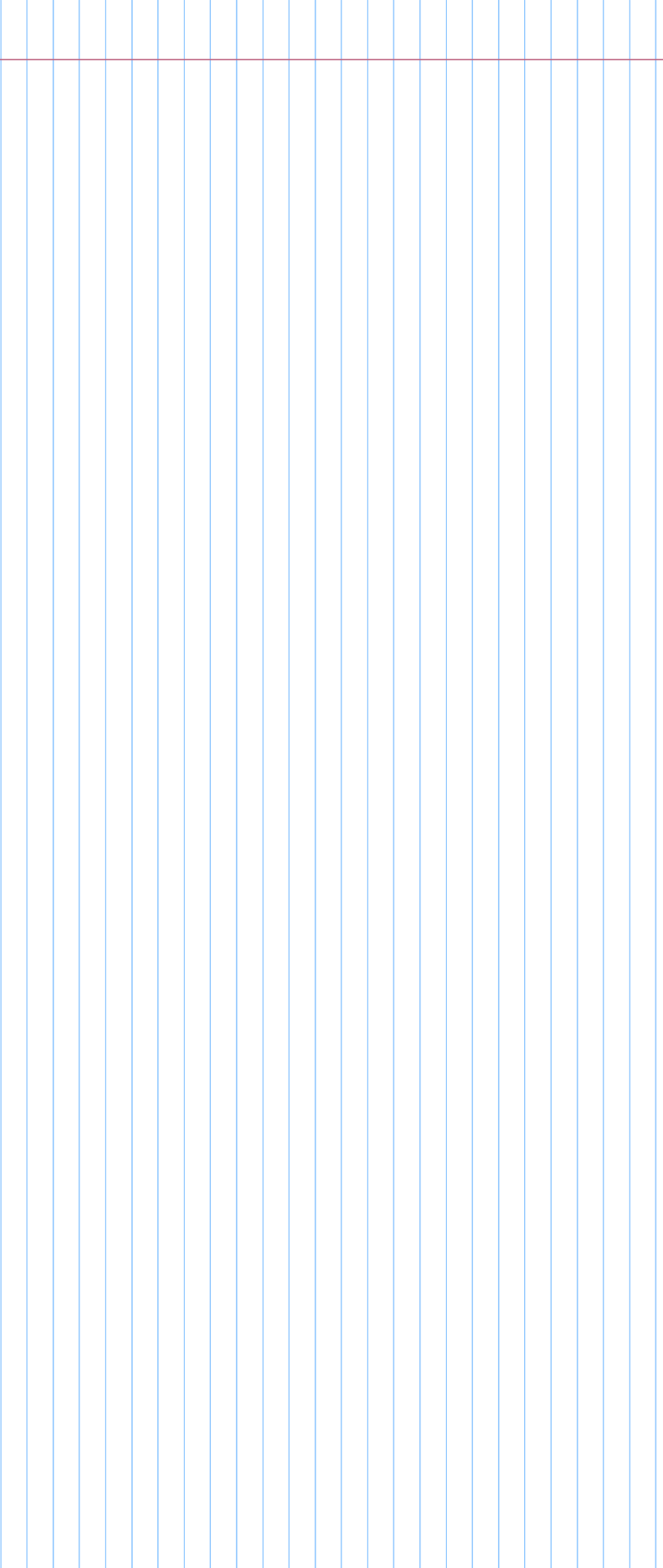
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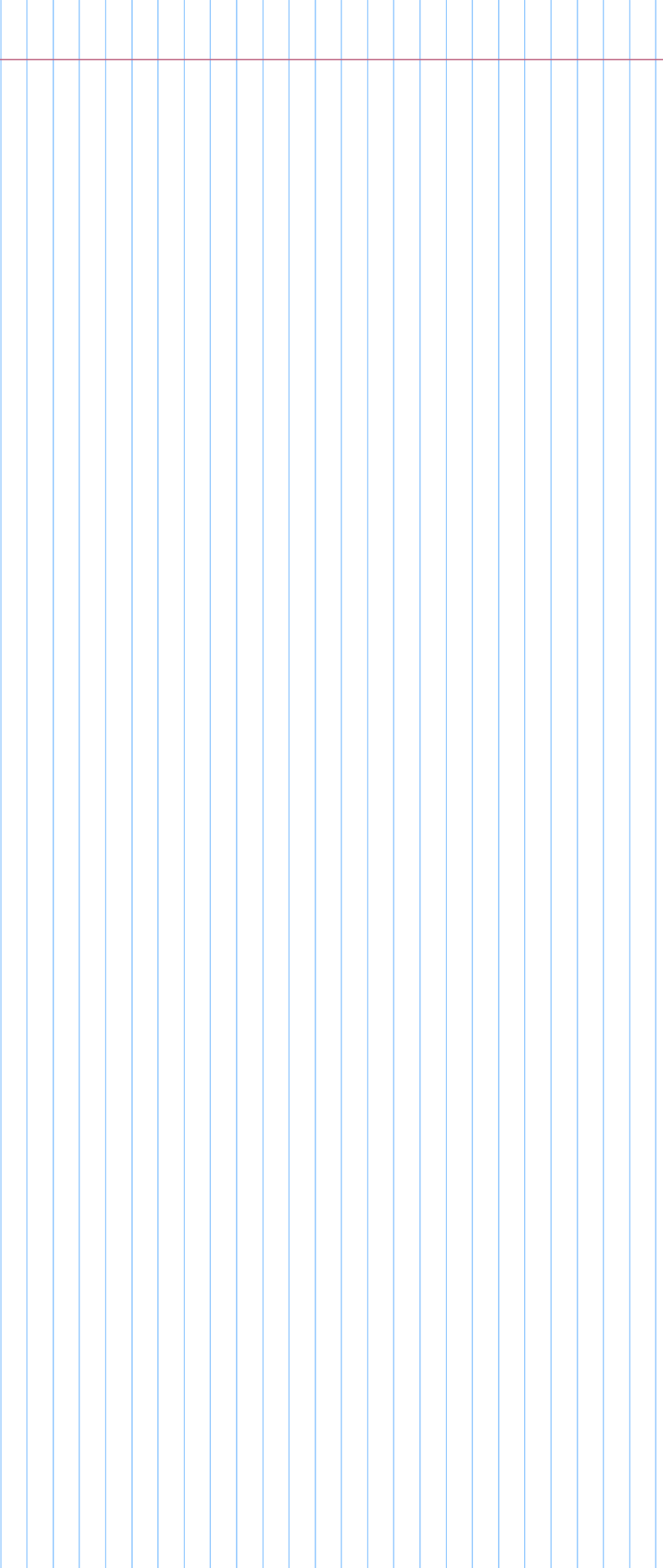




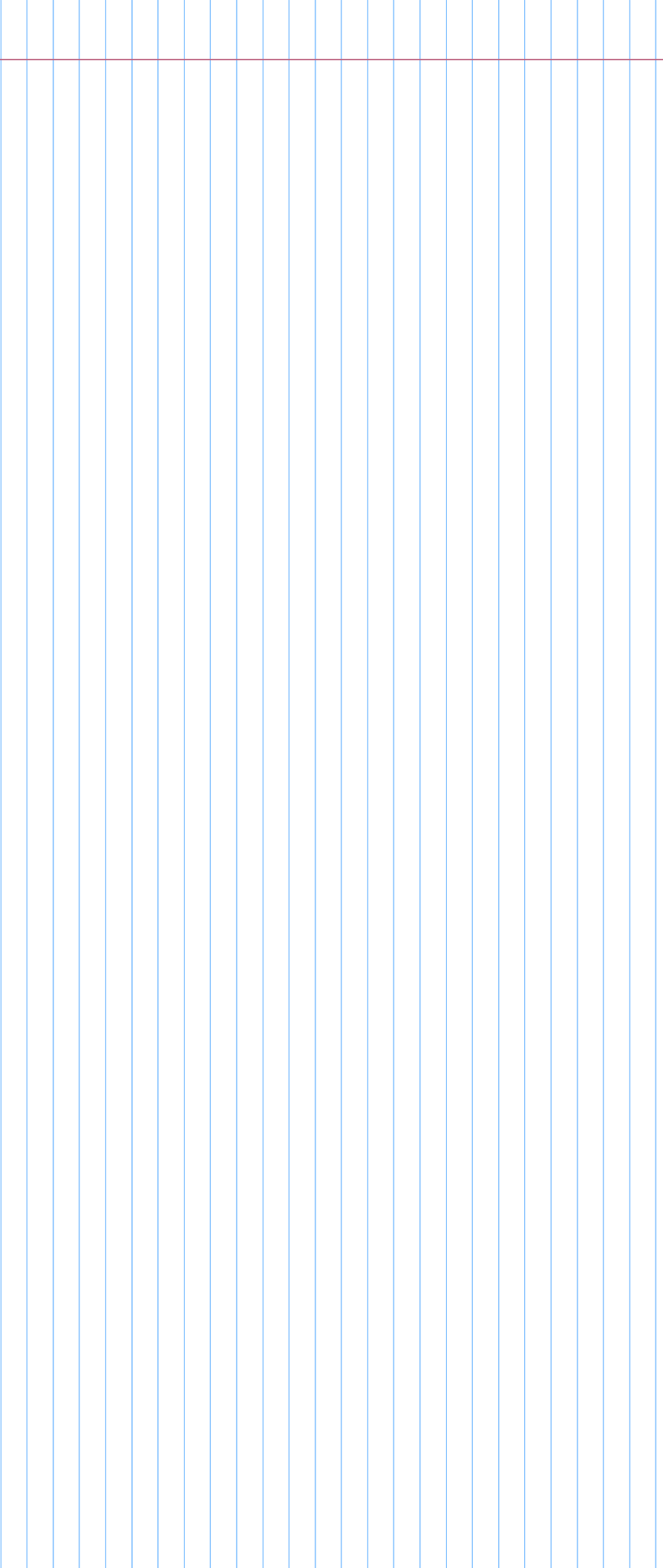














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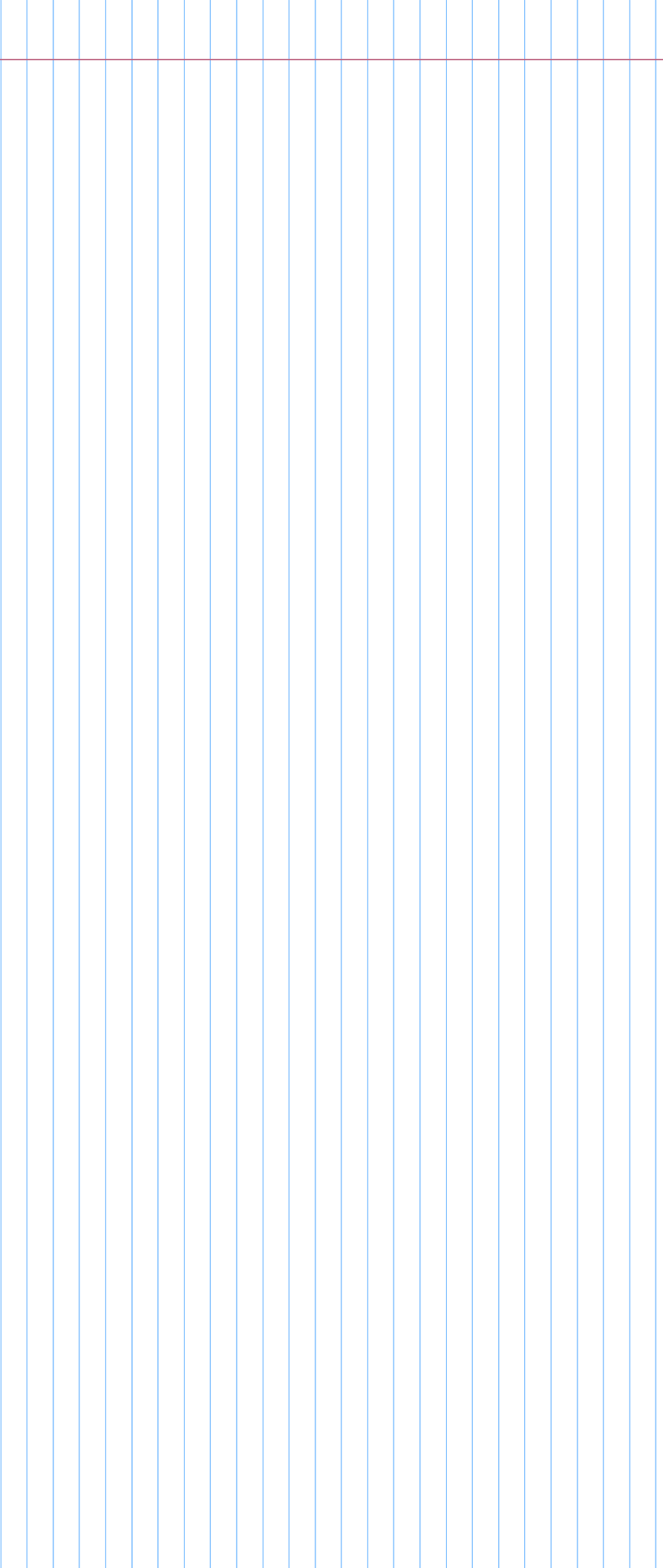
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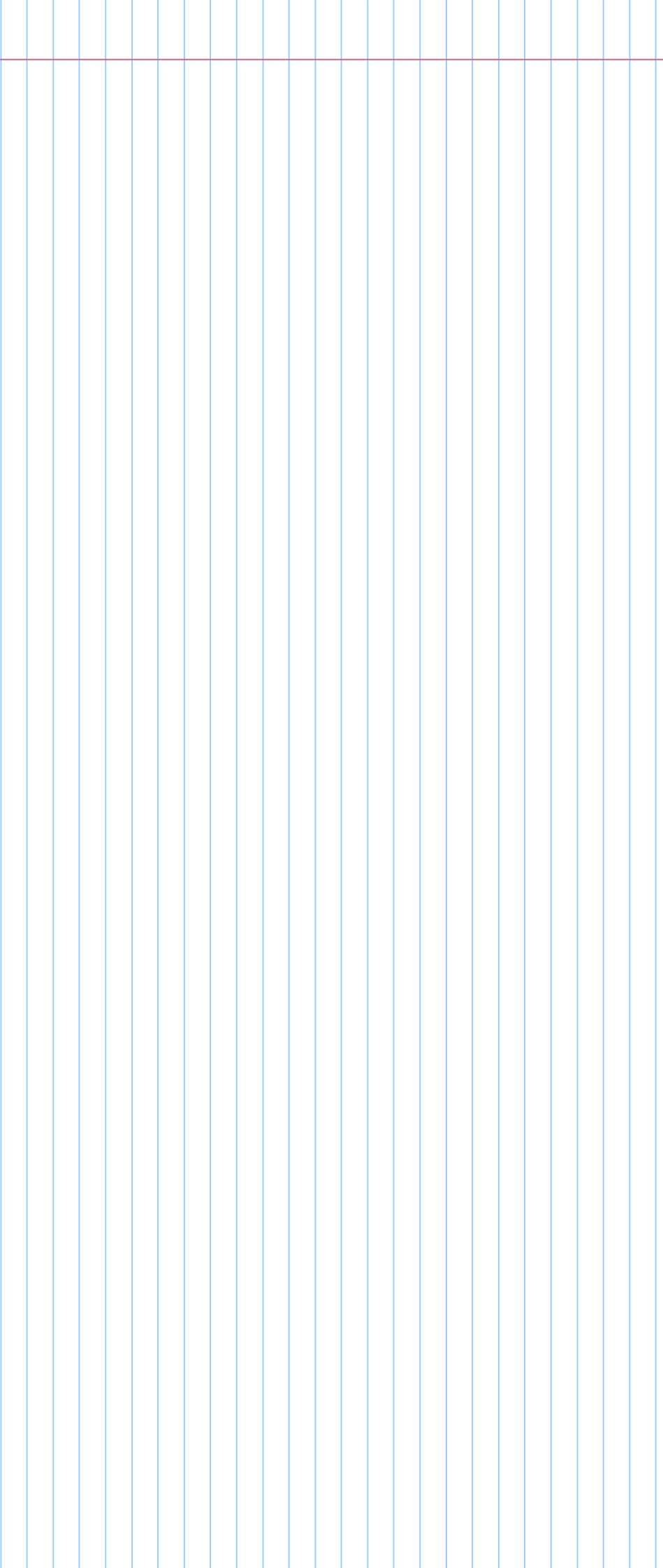


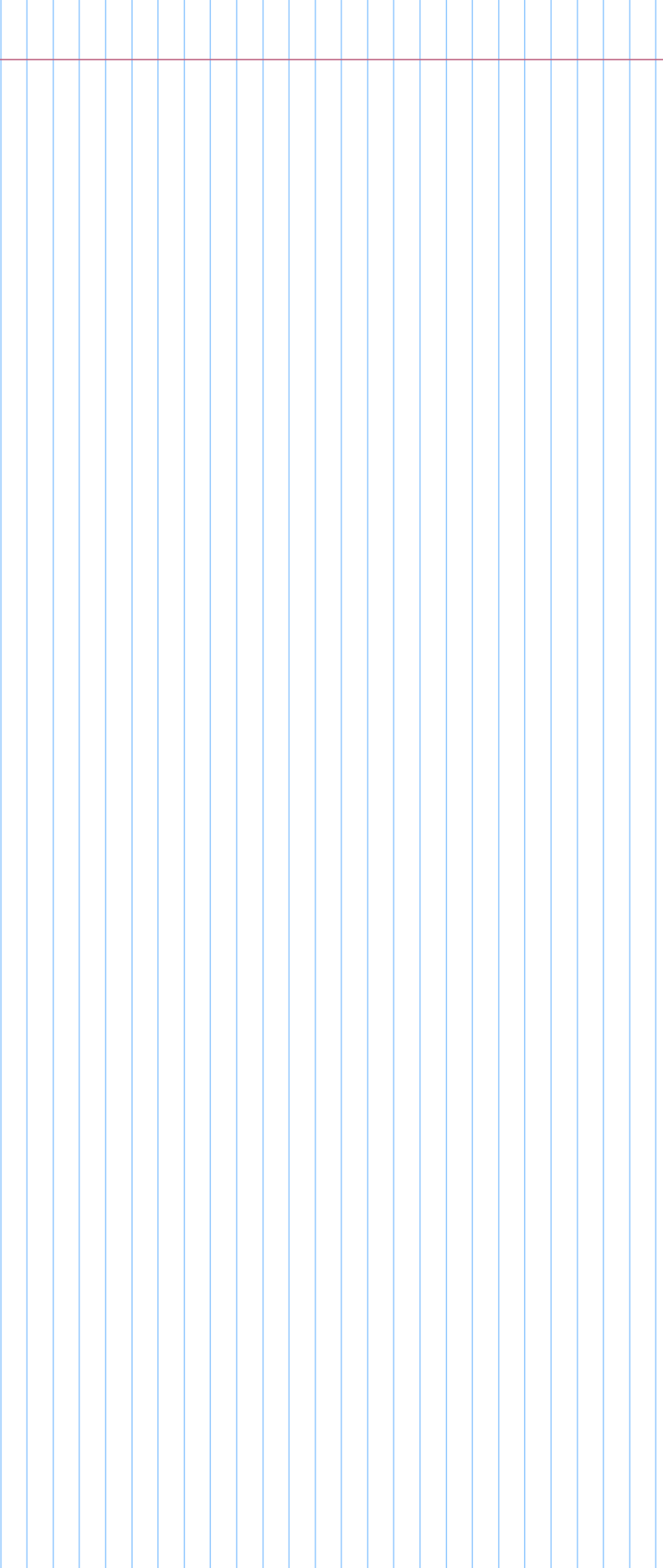
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