

Analysing Branching Algs

Cluster Vertex Deletion - Given $G(V, E)$, are there

k vertices whose deletion results in a cluster graph?

Algm: 1. Find an induced path $x - y - z$ if exists.

If no such P_3 , return Yes ($k \geq 0$)

Otherwise,

if $CVD(a-x, k-1)$

or $CVD(a-y, k-1)$

or $CVD(a-z, k-1)$

return Yes, return Yes

else return No.

If $k=0$ and if there is a P_3 , return No

$$\begin{aligned} (G, k) \\ |V| = n \\ |E| = m \end{aligned}$$

$$\begin{aligned} & \swarrow \quad \downarrow \quad \searrow \\ (k-x, k-1) & \quad (k-y, k-1) & \quad (k-z, k-1) \end{aligned}$$

leaves in the recursion tree for (G, k)
to be $T(m, k)$

$$T(m, k) \leq 3T(m, k-1)$$

or more simply $f(k) \leq 3f(k-1)$

$$f(0) = 1 \quad k$$

$$\text{So } f(k) \approx T(m, k) \leq 3^k$$

Every node has ≥ 2 children
leaves $\approx L$
nodes in the tree

Exercise: G is a tree with all nodes except

the leaves have degree ≥ 3

Then # of nodes in $G \leq 2(\# \text{ leaves})$

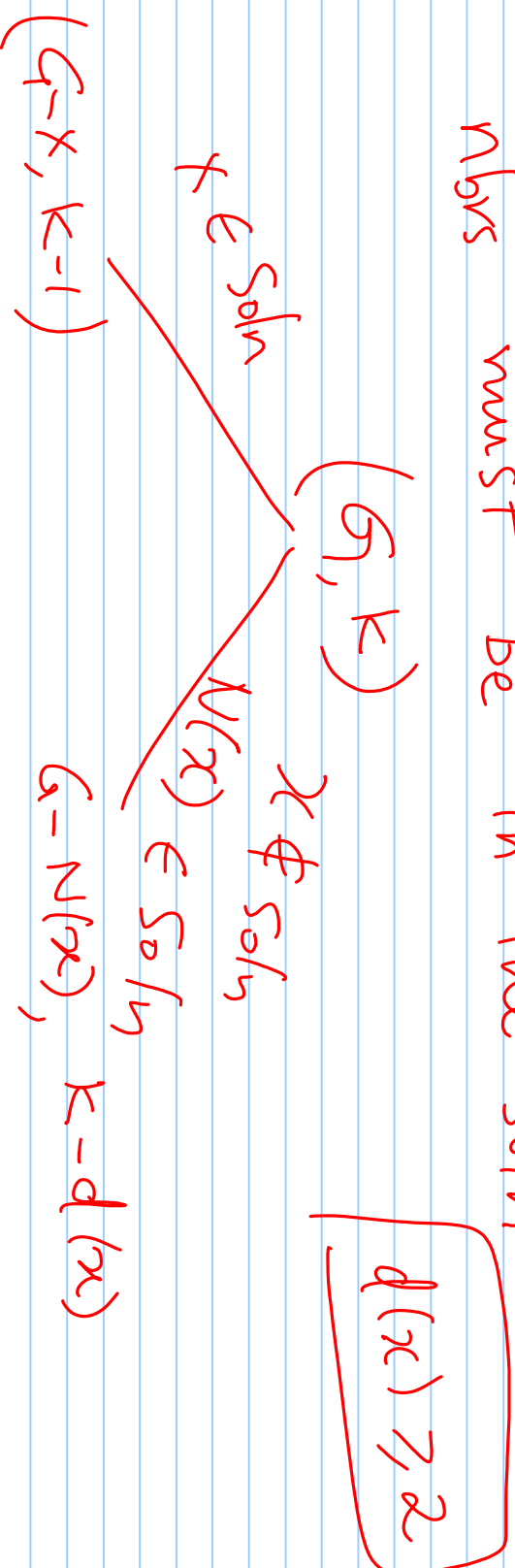
Hence # of nodes in the CVD recursion

$$\text{-tree} \leq 2 \times 3^k$$

So total time for the algm $<$ is $O^*(3^k)$

VC fpt algm $\sim O^*(2^k)$

Obsn: If x is not in the soln, all its
nbrs must be in the soln.



leaves in this tree $T(k) \leq T(k-1) + T(k-2)$

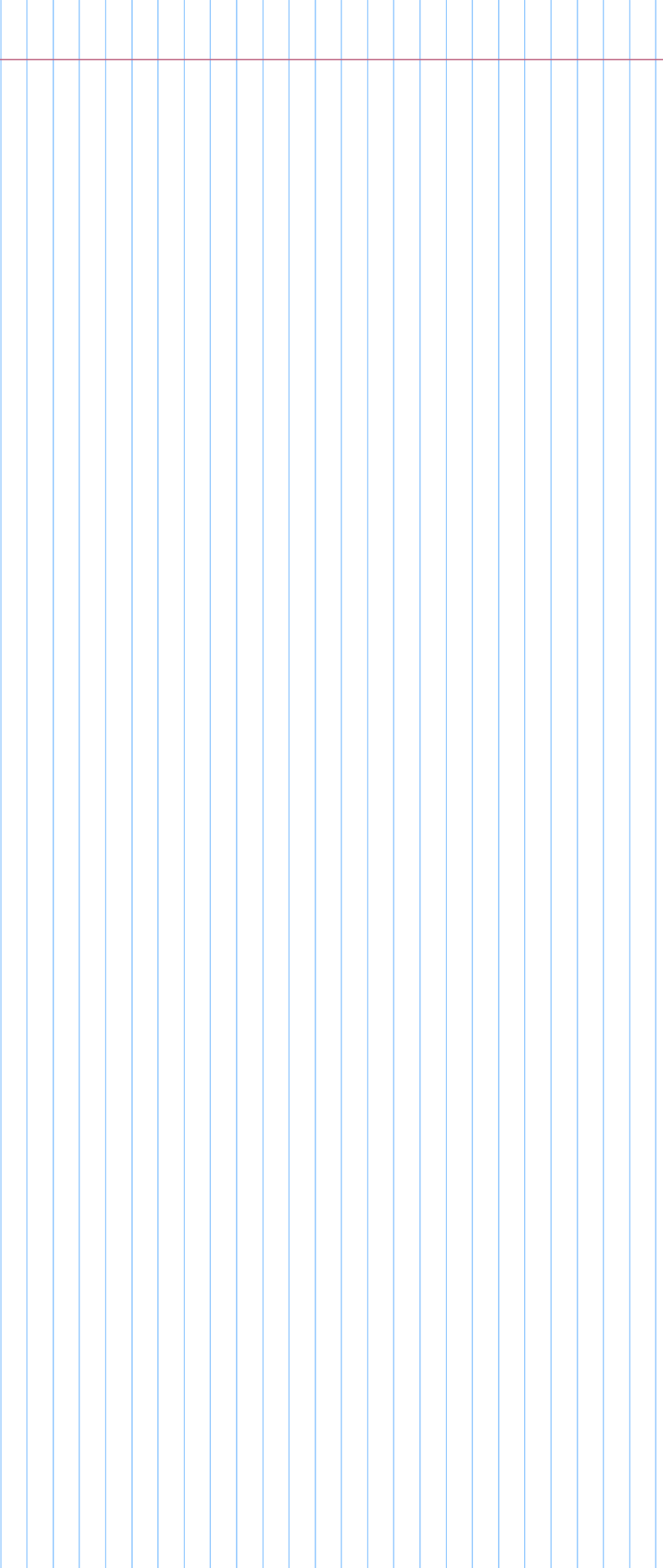
$$\leq T(k-1) + T(k-2)$$

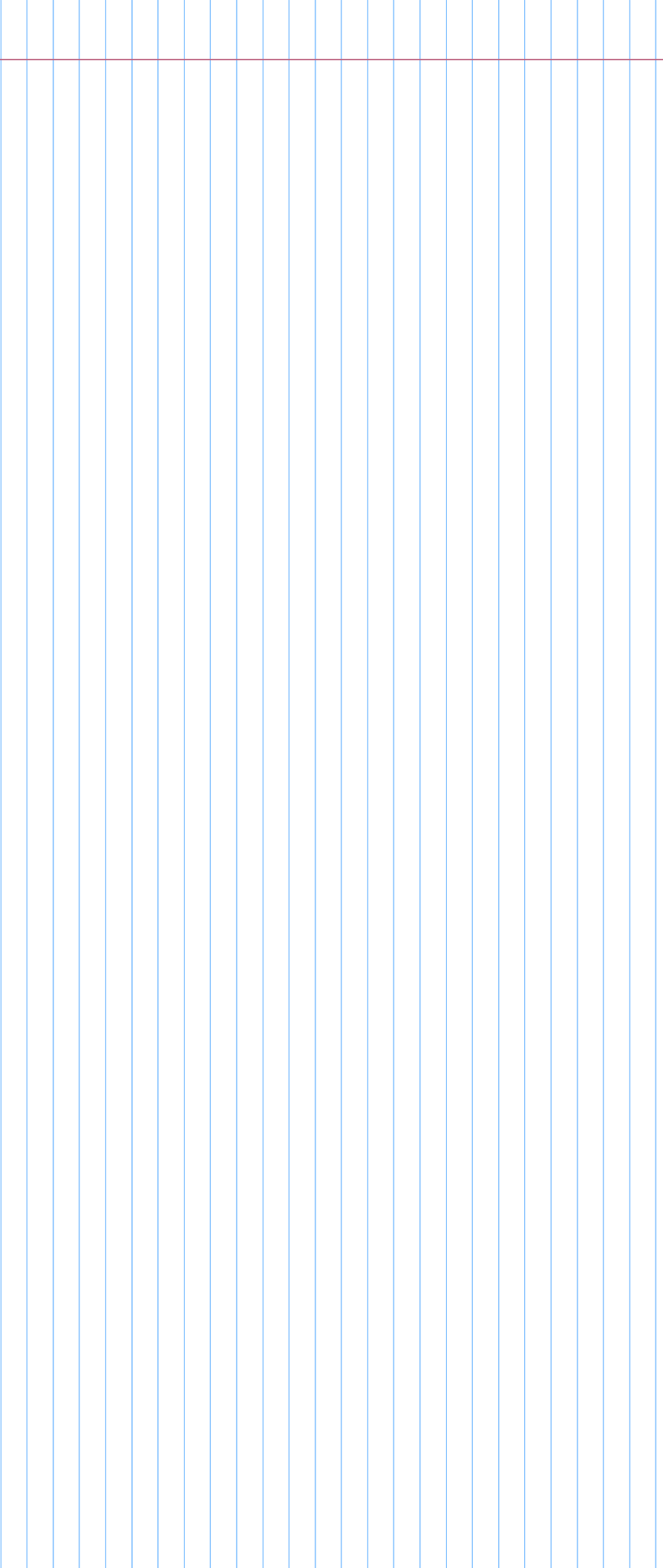
$$\overbrace{C^{k-1} + C^{k-2}} \leq C^k$$
$$C + 1 \leq C^2$$

$$\frac{1+\sqrt{5}}{2} \sim 1.618$$
$$\frac{3.236}{2} \sim 1.618$$

$$O^*(1.618^k)$$

Current best $O^*(1.28^k)$







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