Analysing Branching Algms

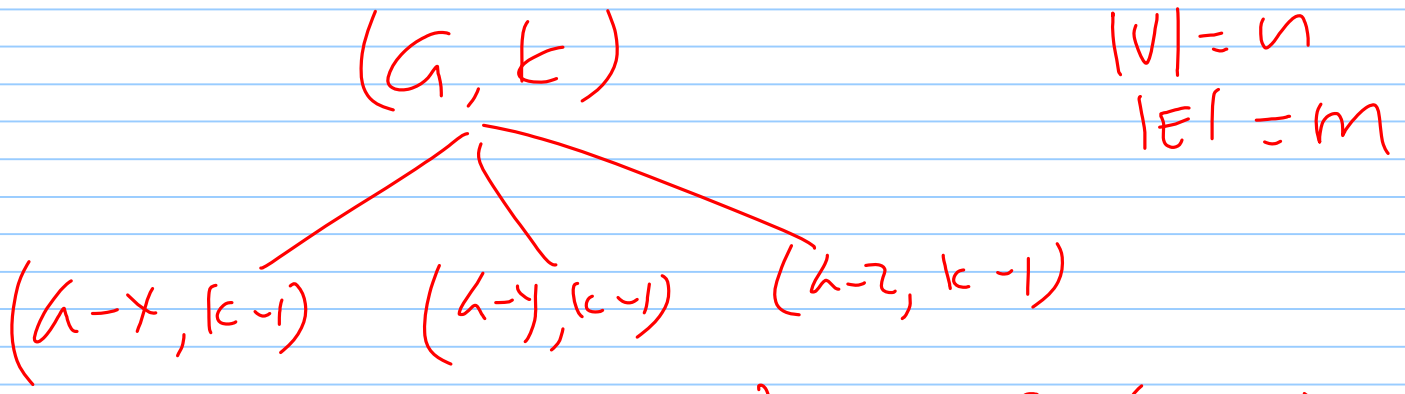
Cluster Vertex Deletion – Given $G(V, E)$, are there k vertices whose deletion results in a cluster graph?

Algm:- CVD(G, k), $k \geq 0$

1. Find an induced path $x - y - z$ if exists.
   If no such P_3, return Yes ($k \geq 0$)
   Otherwise,

if CVD(G-x, k-1)
or CVD(G-y, k-1)
or CVD(G-z, k-1)
return yes, return yes
else return No.

If k=0 and if there is a P_3, return No



$|V| = n$

$|E| = m$

leaves in the recursion tree for (G, k) to be $T(m, k)$

$$T(m,k) \leq 3T(m,k-1)$$

or more simply $f(k) \leq 3f(k-1)$

$f(0) = 1$

So $f(k) \sim T(m,k) \leq 3^k$

Every node has ≥ 2 children

leaves $\sim \ell$

nodes in the tree

Exercise: $\mathcal{T}$ is a tree with all nodes except the leaves have degree ≥ 3

Then # of nodes in $\mathcal{T} \leq 2(\#\text{ leaves})$

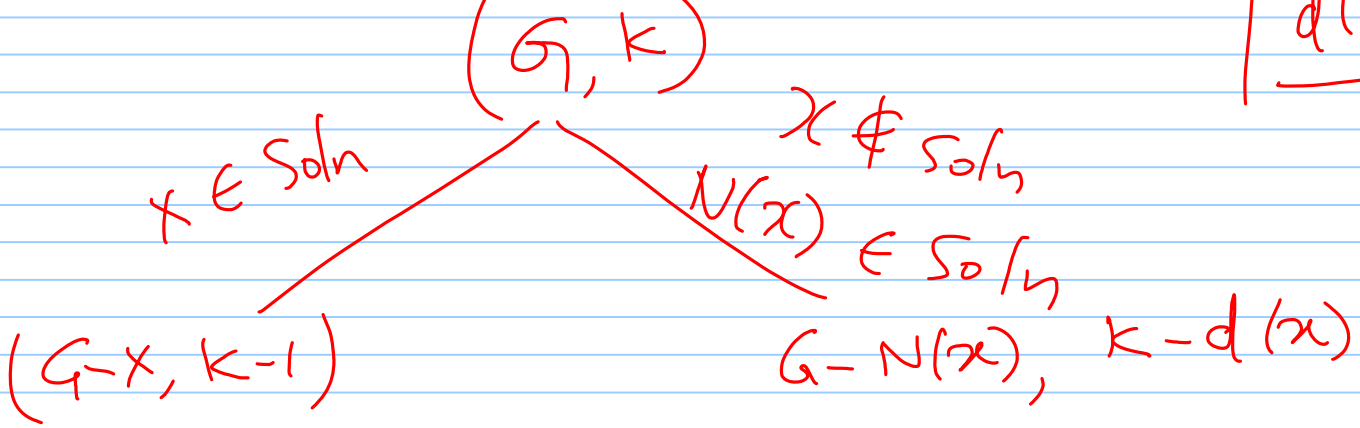
Hence # of nodes in the CVD recursion-tree $\leq 2 \times 3^k$

So total time for the algm $< \text{is } O^*(3^k)$

VC fpt algm $\sim O^*(2^k)$

Obsn: If x is not in the soln, all its nbrs must be in the soln.

$d(x) \geq 2$



leaves in this tree $T(k) \leq T(k-1) + T(k-d(x))$

$\leq T(k-1) + T(k-2)$

$c^{k-1} + c^{k-2} \leq c^k$

$c + 1 \leq c^2$

$\frac{1+\sqrt{5}}{2} \sim \frac{3.236}{2} \sim 1.618$

$O^*(1.618^k)$

Current best $O^*(1.28^k)$