Data Structures **Test II Part II** **Max Marks: 15**

1. Suppose a binary search tree with n nodes has height (number of nodes in the path from the root to a farthest leaf node) h. What is the maximum value for n? (as a function of h). justify. [2]

   Soln: $2^h - 1$.

2. Let $int\ Heap[40]$ be used for dynamic memory allocation with $Alloc()$ and $Free()$ as discussed in the class. Assume that block size is 8 and $int\ head$ holds the index of the first free block. [2+2]

   a) Write down the contents of $head$, $Heap[0]$, $Heap[8]$, $Heap[16]$, $Heap[24]$ and $Heap[32]$. After the following: $Initialize(); int\ x = Alloc(); int\ y = Alloc(); int\ z = Alloc(); Free(x); Free(z);$

   Soln: $head = 16, Heap[0] = 24, Heap[16] = 0, Heap[24] = 32, Heap[32] = -1$.

   b) Write code for the $Free()$ function. (Working C code is required).

   Soln:

```
void Free(int p} {
     Heap[p]=head;
     head=p;
     return;
}
```

3. Suppose we want to find the **average height** of a randomly built binary search tree, built by random insertions from $\{1, 2, .., n\}$. Let X_n denote a random variable for height of a randomly constructed tree with n nodes. **Formulate a recurrance** for $E(X_n)$, clearly justifying your steps. (Hint: It may contain expressions like $max(E(X_i), E(X_j)))$. Justify your recurrance model. Assuming that $E(X_i) \leq E(X_j)$ if $i \leq j$. Reformulate the recurrance eliminating the max function, and having only arithmetic operations (the expression will give an upper bound to the expected value). Justify the assumtions. (You don't have to solve the recurrance. Marks will be given only if a) Your formulation is justifiable and your bound is not too weak to derive a correct expected height estimate). [4]

   Soln: Let X_n be a random variable indicating the height of a randomly constructed BST. Let Y be the value of the first element inserted. $Pr(Y = i) = \frac{1}{n}$ for $1 \leq i \leq n$. $E(X_n|Y = i) = E(1+\max\{X_{i-1}, X_{n-i}\} = 1+E(\max X_{i-1}, X_{n-1})$. This yields $E(X_n) = \sum_{i=1}^{i=n} \frac{1}{n}(E(X_n|Y = i) = 1 + \frac{1}{n} \sum_{i=1}^{i=n} E(\max\{X_{i-1}, X_{n-i}\})$. Full marks will be given if you reach upto this.

4. This solution key to this question is deliberately left unpublished. [5]