MFCS 2017 Mock Test Questions

1. Suppose T be a linear operator on $\mathbf{R}^3$ such that $[1, 1, -1]^T, [1, -1, 1]^T$ and $[-1, 1, 1]^T$ are Eigen vectors of T with Eigen values $1, -1$ and 0. What is the matrix of T with respect to the standard basis? [3]

2. Let T be a linear transformation from V to W, where V, W are vector spaces over some field F. Let $v_0 \in V$. Show that for any $v \in V$, $T(v) = T(v_0)$ if and only if $v = v_0 + w$ for some $w \in ker(T)$. [3]

3. Let $(R, +, ., 0, 1)$ be a finite integral domain. Show that for any $a \in R \setminus \{0\}$, there exists $b \in R$ such that $a.b = 1$. [3]

4. Let V, W be vector spaces over a field F. Let $T : V \mapsto W$ be a linear transformation. Let $b_1, b_2, \cdots b_n$ be a basis of V. Prove that [3+3]

   1. $T(b_1), T(b_2), \cdots, T(b_n)$ is linearly independent if and only if T is injective.
   2. SPAN$(T(b_1), T(b_2), \cdots, T(b_n)) = W$ if and only if T is surjective.

   (Answer on the reverse side).