Computational Complexity **Exercise 1I**

1. Recall that CNFSAT is the decision problem of determining whether a boolean formula in conjunctive normal form is satisfiable. The 3SAT problem puts an additional constraint that the formula is 3 conjunctive normal form - i.e., each clause contains at most 3 literals. Show that CNFSAT $\preceq_m^p$ 3SAT. □

2. The CLIQUE/VC/IS problem takes as input a graph $G(V, E)$ and a number k and asks whether G contains a clique/vertex cover/independent set of size k. Show that 3SAT$\preceq_m^p$CLIQUE$\preceq_m^p$VC$\preceq_m^p$IS. □

3. Reading Assignment: Read the chapter on NP completeness from Sipser's book. Read the section on PSPACE completeness from Sipser's book. Study the proof that Generalized Geography problem is PSPACE complete. □

4. A directed graph G is strongly connected if there is a (directed) path from every vertex to every other vertex in G. The problem SCONN takes as input a graph G and asks whether G is strongly connected. Show that the problem is NL complete. (Reduce from s-tREACH). □

5. Show that the problem of deciding whether a graph is bipartite is in NL. (A bit of graph theory reading bipartite graphs will be needed - if you are non-CS) □

6. Show that NC$_1$ $\subseteq$ L. □

7. Let $s(n) \geq \log n$ be space constructible and $t(n) \geq n$ be time constructible. Define UDepth($s(n)$) as the set of languages for which uniform circuits of $O(s(n))$ depth exists. Similarly USize($t(n)$) is the class of languages for which uniform circuits of size $O(t(n))$ exists. □

   - Show that NSPACE($s(n)$) $\subseteq$ Udepth($s^2(n)$).
   - UDepth($s(n)$) $\subseteq$ DSPACE($s(n)$).
   - DTIME($t(n)$) $\subseteq$ USize($t^2(n)$.
   - USize($t(n)$) $\subseteq$ DTIME($t(n)$.

8. Show that if NC$_{i+1}$ $\subseteq$ NC$_i$ for some $i \geq 2$ then NC=NC$_i$. What conclusion can be made about AC from the above? Show that if a P-complete problem is in NC, then NC=NC$_i$ for some i. □

9. If PH has a complete language L, then PSPACE=PH. □

10. Suppose we define the following hierarchy Σ_1^L =NL, Σ_2^L=NL^{NL}, .. Σ_{i+1}^L=NL$^{\Sigma_i^L}$. Show that the hierarchy collapses to NL. □

11. Reading Assignment: Read the section of **alternating Turing machines** from Sipser's book. Prove that ATIME($t(n)$) $\subseteq$ DSPACE($t(n)$) $\subseteq$ ATIME($t^2((n))$ for $t(n) \geq n$ time constructible. For $s(n) \geq \log n$ space constructible, prove that ASPACE($s(n)$) $\subseteq$ DTIME($2^{O(s(n))}$) $\subseteq$ ASPACE($s(n)$). Show that Σ_i^p and Π_i^p presented in the book are equivalent to the definitions presented in the class. □